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LORENTZIAN ISOPARAMETRIC HYPERSURFACES

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A Lorentzian hypersurface will be called isoparametric if the minimal polynomial of the shape operator is constant. This allows for complex or non-simple principal curvatures (eigenvalues of the shape operator). This paper locally classifies isoparametric hypersurfaces in Lorentz space.

The classification is done by proving Cartan-type identities for the principal curvatures and showing that the hypersurface can have at most one non-zero real principal curvature. Standard examples are given in §3 and the main theorems are in §4.

The hypersurfaces with minimal polynomials $(x - a)^2$ and $(x - a)^3$ are called generalized umbilical hypersurfaces since they have exactly one principal curvature. The classification of these hypersurfaces gives insight into principal curvatures and the effect of the constant principal curvatures on the structure of a hypersurface.

1. Preliminaries. In this paper all manifolds and maps are assumed to be C^∞. $f\colon M \to \tilde{M}$ will always be an immersion but f can be treated locally as an embedding. Thus x will often be identified with $f(x)$ and the mention of f will be supressed.

Lorentz space and its hypersurfaces. Let $\mathbf{L}^{n+1}$ be the $n+1$ dimensional real vector space $\mathbf{R}^{n+1}$ with an inner product of signature $(1, n)$ given by

$$(\vec{x}, \vec{y}) = -x_0y_0 + \sum_{i=1}^{n} x_iy_i$$

for $\vec{x} = (x_0, x_1, \ldots, x_n)$ and $\vec{y} = (y_0, y_1, \ldots, y_n)$. $\mathbf{L}^{n+1}$ is called Lorentz space.

The n-dimensional sphere of radius r in $\mathbf{L}^{n+1}$, $S_1^n(1/r^2)$, is the hypersurface

$$\{\vec{x} \in \mathbf{L}^{n+1}\colon (\vec{x}, \vec{x}) = r^2\}$$

with the induced Lorentzian metric. It has constant sectional curvature $1/r^2$.

Generally a hypersurface M in $\mathbf{L}^{n+1}$ is called a Lorentzian hypersurface if the induced metric has signature $(1, n-1)$. If D is the flat connection on $\mathbf{L}^{n+1}$ the Levi-Civita connection ∇ on M is specified by

$$D_XY = \nabla_XY + \alpha(X, Y) \tag{1.1}$$