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REDUCIBILITY OF POLYNOMIALS IN SEVERAL VARIABLES. II

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In Loving Memory of Ernst G. Straus

Let $f_i(x_i)$ be non-constant rational functions over a field K ($i = 1, 2, \ldots, n$). A necessary and sufficient condition is given for reducibility over K of the numerator of the sum $\sum_{i=1}^{n} f_i(x_i)$ in its reduced form, provided $n \geq 3$. In particular the numerator is irreducible if $\operatorname{char} K = 0$, which generalizes a theorem of Ehrenfeucht and Pełczyński and answers a question of M. Jarden.

A. Ehrenfeucht and A. Pełczyński answering a question of A. Mostowski have proved that a polynomial

$$F(x) + G(y) + H(z),$$

where F, G, H are nonconstant polynomials over the complex field C is irreducible over C. For the proof which extends to all fields of characteristic zero see [3] or [5]. In [4] (p. 53) the following generalization to fields of arbitrary characteristic has been proved. Let K be a field and $F, G, H \in K[x] \setminus K$. Then $F(x) + G(y) + H(z)$ is reducible over K if and only if

$$F(x) - F(0) = L(F_1(x)), \qquad G(y) - G(0) = L(G_1(y)),$$

$$H(z) - H(0) = L(H_1(z)),$$

where $L \in K[t]$ is an additive polynomial, $F_1 \in K[x]$, $G_1 \in K[y]$, $H_1 \in K[z]$ and $L(t) + F(0) + G(0) + H(0)$ is reducible over K.

Let us adopt the following

DEFINITION 1. A rational function is reducible over K if the numerator in its reduced form is reducible over K.

Recently M. Jarden has asked whether with this definition polynomials in Ehrenfeucht and Pełczyński's theorem can be replaced by rational functions. We shall answer this question in the affirmative by proving a more general result concerning fields of arbitrary characteristic. This generalizes also the result quoted above. To formulate it we need some notation.