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MOMENTS OF ADDITIVE FUNCTIONS AND THE SEQUENCE OF SHIFTED PRIMES

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Dedicated to the memory of Ernst Straus, my teacher

Recently, by means of a new method involving the combinatorial sieve and the bilateral Laplace transform, we estimated asymptotically the moments of additive functions $f(n)$ for integers n belonging to certain sets S. From such estimates the limiting distribution function of these $f(n)$, for $n \in S$, can be determined. Here the method is applied to the special sequence $S_c = \{p + c\}$, where p runs through all the primes and c is an arbitrary fixed integer. Various distribution properties of the sequence S_c, such as those given by the Brun-Titchmarch inequality and Bombieri's theorem, are used. Previously Barban had established distribution results for certain $f(n)$ when $n \in S_c$, but it was not known (until now) under what conditions the moments could be asymptotically estimated as well.

1. Introduction, notation and main result. In a recent paper [**1**] I used a new method to asymptotically estimate the moments of certain additive functions $f(n)$ for integers belonging to some special sets S. The method is based upon the combinatorial sieve and certain properties of the bilateral Laplace transform. It therefore has advantages over some earlier approaches in two respects. First, the use of the sieve enables one to treat a fairly large class of sets S. Next, the bilateral Laplace transform introduces simplification in the calculation of moments. For a detailed description of the method see [**1**], whereas for a comparative study of this technique against a classical background see [**2**].

We shall now investigate the applciations of this method to the set S_c of shifted primes given by

$$S_c = \{ p + c | p = 2, 3, 5, \dots, \text{primes} \}, \tag{1.1}$$

where c is any fixed positive integer. Our goal is to prove Theorem 1 below. In doing so we shall come across many interesting auxiliary results. But first we need some notation.

Additive functions f are arithmetical functions satisfying $f(m \cdot n) = f(m) + f(n)$, whenever g.c.d.$(m, n) = 1$. For simplicity we concentrate