PACIFIC JOURNAL OF MATHEMATICS
Vol 126, No. 2, 1987

A NOTE ON THE FUNDAMENTAL GROUP OF A COMPACT MINIMAL HYPERSURFACE

GREGORY J. GALLOWAY

In this paper we generalize a well-known result of Frankel which relates the fundamental group of a complete Riemannian manifold with positive Ricci curvature to the fundamental group of a compact immersed minimal hypersurface. Here we consider the situation in which the Ricci curvature of the ambient manifold is only assumed to be nonnegative, and show that the conclusion of Frankel's theorem can fail only under special circumstances.

1. Introduction. The theory of minimal surfaces has provided a powerful tool for studying the topology of complete Riemannian manifolds of low dimension with nonnegative scalar or Ricci curvature. In general, it is a problem of basic interest to study the topological and geometrical relationships between a minimal submanifold and the manifold in which it is immersed. A well-known result of Frankel [**1**] asserts that if Σ is a compact immersed minimal hypersurface in a Riemannian manifold M with strictly positive Ricci curvature then the homomorphism of fundamental groups: $\Pi(\Sigma) \to \Pi(M)$ induced by inclusion is onto. As the product of spheres $S^1 \times S^2$ shows, the conclusion of Frankel's theorem is false if the Ricci curvature is only assumed to be nonnegative. The purpose of this paper is to study the rigidity of Frankel's theorem, i.e. to study the extent to which Frankel's theorem can fail when the Ricci curvature is only assumed to be nonnegative. Our main theorem, stated below, shows that the theorem can fail only under special circumstances.

THEOREM. *Let M be a complete n-dimensional Riemannian manifold with nonnegative Ricci curvature. Let $\varphi\colon \Lambda \to M$ be a minimal immersion, where Λ is a compact $(n-1)$-dimensional manifold, and let $\Sigma = \varphi(\Lambda)$. Consider the homomorphism of fundamental groups $i_*\colon \Pi(\Sigma) \to \Pi(M)$ induced by the inclusion map $i\colon \Sigma \to M$. Then either* (a) *below holds, or Σ is an imbedded totally geodesic submanifold of M and one of* (b)–(e) *holds.*

(a) *i_* is onto.*

(b) $\Pi(M)/i_*(\Pi(\Sigma)) = Z_2$. *$\Sigma$ separates M, and the closure of one of the components of $M - \Sigma$ has a double covering which is isometric to* $[0, L] \times \Sigma$.