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THE DUAL PAIR ($U(3)$, $U(1)$) OVER A p-ADIC FIELD

COURTNEY MOEN

This paper considers some aspects of the oscillator representation of the dual reductive pair $(U(3), U(1))$ over a p-adic field, with p odd.

1. Introduction. One of the simplest examples of a dual pair arising from Howe's general construction [**Ho**] is that of $(U(3), U(1))$. We will study this pair in the p-adic case, with $p \neq 2$. We first give the details of what the construction provides in this case and review some necessary results from [**M**] concerning the dual pair $(U(1), U(1))$. We then consider the irreducible constituents of the oscillator representation restricted to $U(3)$. We first determine which of these constituents embed in principal series, and we find some explicit information concerning these embeddings. We next show that each irreducible supercuspidal constituent is induced from a representation of a maximal compact subgroup of $U(3)$. A surprising feature is that in all cases except that in which $U(3)$ is defined over an unramified extension and we are considering representations of conductor one, the group over the ring of integers does not suffice, and we must use the other class of maximal compact subgroups.

The results in this paper concerning principal series were discovered originally by Howe and Piatetskiĭ-Shapiro and appear in [**GPS**], where they play a role in some of the authors' important results concerning automorphic forms in $U(3)$. The methods of this paper borrow heavily from [**A**] and [**Ho**]. I would also like to thank C. Asmuth for a useful conversation.

2. Basic construction. Let F be a p-adic field, with $p \neq 2$. Let $\mathcal{O}$ be the ring of integers, P the prime ideal, U the units, ν the additive valuation, and π a prime element. $E = F(\sqrt{\alpha})$ will be a quadratic extension of F, with $\mathcal{O}_E$, P_E, U_E, ν_E, and π_E the corresponding objects for E. Let q be the order of $\mathcal{O}/P$.

Let h_1 be the 3-dimensional Hermitian form over E defined by

$$h_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Let h_2 be the 1-dimensional form defined by $h_2(x, y) = xy^\sigma$, where $y \to \bar{y} = y^\sigma$ is the Galois action of E/F. Let $U(3)$ and $U(1)$ be the associated unitary groups.