PACIFIC JOURNAL OF MATHEMATICS
Vol. 142, No. 2, 1990

EXAMPLES OF FOLIATIONS WITH FOLIATED GEOMETRIC STRUCTURES

LUIS A. CORDERO AND ROBERT A. WOLAK

We present examples of foliated compact nilmanifolds, whose foliations are neither simple nor given by suspensions, admitting various foliated geometric structures. For example, we construct foliations which are:
1. transversely symplectic but not transversely Kähler,
2. transversely symplectic but not transversely holomorphic,
3. transversely Sasakian but not transversely cosymplectic.

1. Foliated structures on foliated manifolds. Let $(M, \mathscr{F})$ be a foliated manifold. The foliation $\mathscr{F}$ is called an (N, G)-structure (cf. [**14**]) if $\mathscr{F}$ is given by a cocycle $\mathscr{U} = \{U_i, f_i, g_{ij}\}$ where:

1. $\{U_i\}$ is an open covering of M,
2. $f_i\colon U_i \to N$ are submersions with connected fibres,
3. $g_{ij}\colon f_j(U_i \cap U_j) \to f_j(U_i \cap U_j)$ are local diffeomorphisms of N for which
   (a) $f_i|_{U_i \cap U_j} = g_{ij} \circ f_j|_{U_i \cap U_j}$,
   (b) there exists $h_{ij} \in G$ such that $h_{ij}|_{f_i(U_i \cap U_j)} = g_{ij}$.

If the group G acts quasi-analytically (i.e. if for some element h of G there exists an open subset of N on which h is the identity transformation then h itself is the identity), then we have the following

LEMMA (*Thurston* [14]). *If the group G acts quasi-analytically, then any (N, G)-structure is developable, i.e. there exists a covering $\widehat{M}$ of M and a developing mapping $D\colon \widehat{M} \to M$ such that the lifted foliation $\widehat{\mathscr{F}}$ is given by the fibres of D. Moreover, there is a homomorphism $h\colon \pi_1(M) \to G$, called the holonomy homomorphism, such that the mapping D is $\pi_1(M)$-equivariant for these two actions of the group.*

Any foliated geometric structure of $(M, \mathscr{F})$ defines the corresponding structure on the transverse manifold (cf. [**16**, **17**]), which in this case can be chosen to be an open submanifold of N. If the developing mapping is surjective and its fibres are connected, then the transverse manifold can be identified with N. The holonomy pseudogroup of $(M, \mathscr{F})$ has as its representative a pseudogroup generated