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DIFFERENTIAL GEOMETRY OF SYSTEMS OF PROJECTIONS IN BANACH ALGEBRAS

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Let A be a Banach algebra, n a positive integer and $Q_n = \{(q_1, \dots, q_n) \in A^n: q_i q_k = \delta_{ik} q_i,\ q_1 + \cdots + q_n = 1\}$. The differential geometry of Q_n, as a discrete union of homogeneous spaces of the group G of units of A is studied, a connection on the principal bundle $G \to Q_n$ is defined and invariants of the associated connection on the tangent bundle TQ_n are determined.

Introduction. The structure of the set Q of all idempotent elements of a Banach algebra A plays a fundamental role in several aspects of spectral theory. This work deals with the differential structure of the space

$$Q_n = \left\{(q_1, \dots, q_n) \in A^n: q_i q_k = \delta_{ik} q_i, \sum_{i=1}^{n} q_i = 1\right\}$$

of systems of n "orthogonal" projections in A.

The manifold Q_n appears as a universal model when certain polynomial equations are considered. More precisely, if $\alpha_1, \dots, \alpha_n$ are *different* complex numbers and $\alpha(X)$ denotes the polynomial $(X - \alpha_1)\cdots(X - \alpha_n)$, then the set $A_\alpha = \{a \in A: \alpha(a) = 0\}$ is a closed submanifold which is diffeomorphic to Q_n. Thus Q_n is the model for all simple algebraic elements of A of degree n. Moreover, Q_n plays a role in the study of arbitrary algebraic (in particular, nilpotent) elements (see **[AS]**).

Section 1 contains the description of the differential structure of Q_n and A_α as closed analytic submanifolds of A^n and A, respectively; it contains also the proof that Q_n and A_α are diffeomorphic.

Using Kaplansky's notion of SBI-rings, we recover a result of Barnes **[Ba]** concerning the surjectivity of $A_\alpha \to B_\alpha$ when B is the quotient of A by its Jacobson radical. In §2 we show that Q_n is a discrete union of homogeneous spaces of G, the group of units of A; this fact, together with a classical result of Michael **[Mi]**, shows that an epimorphism $f: A \to B$ of Banach algebras induces Serre fibrations $Q_n(A) \to Q_n(B)$ and $A_\alpha \to B_\alpha$. In §3 we obtain an explicit way of