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AN ALMOST CLASSIFICATION OF COMPACT LIE GROUPS WITH BORSUK-ULAM PROPERTIES

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We say that a compact Lie group G has the Borsuk-Ulam property in the weak sense if for every orthogonal representation V of G and every G-equivariant map $f\colon S(V) \to S(V)$, $V^G = \{0\}$, of the unit sphere we have $\deg f \neq 0$.

We say that G has the Borsuk-Ulam property in the strong sense if for any two orthogonal representations V, W of G with $\dim W = \dim V$ and $W^G = V^G = \{0\}$ and every G-equivariant map $f\colon S(V) \to S(W)$ of the unit spheres we have $\deg f \neq 0$. In this paper a complete classification, up to isomorphism, of group with the weak Borsuk-Ulam property is given. A classification of groups with the strong Borsuk-Ulam property does not cover nonabelian p-groups with all elements of the order p. In fact we deal with a more general definition admitting a nonempty fixed point set of G on the sphere $S(V)$.

1. The main theorems. In order to formulate our main results we introduce the following notation. Let G be a compact Lie group. We denote by G_0 the component of identity of and by Γ the quotient group G/G_0. We use standard notation of the theory of compact transformation groups (see for instance **[4]** or **[5]**). In particular, for every subgroup $H \subset G$, the fixed point set of H on a G-space X is denoted by X^H. Also, for a G-equivariant map $f\colon X \to Y$ between two G-spaces, we denote by f^H its restriction to the space X^H. The symbol (n, m) stands for the greatest common divisor of the integers n, m with the notation $(0, 1) = 0$ and $|G|$ for the rank of the (finite) group G. We will work with the following definition of the Borsuk-Ulam property (cf. **[11]**).

DEFINITION I. (A) We say that G has the Borsuk-Ulam property in the weak sense A if for every orthogonal representation V of G and every G-equivariant map $f\colon S(V) \to S(V)$ if $(\deg f^G, |\Gamma|) = 1$ then

$$\deg f \neq 0.$$

REMARK. Note that in the case $V^G = \{0\}$ there is no condition on $\deg f^G$ and the property then requires that $\deg f \neq 0$ for every G-equivariant map. Also, if $G = G_0$ then the condition $(\deg f^G, |\Gamma|) = 1$ means that $\deg f^G \neq 0$.