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ON SINGULAR PERTURBATIONS OF SECOND ORDER CAUCHY PROBLEMS

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We give an explicit formula for the solution of complete second order Cauchy problems in Banach spaces. Using this formula we derive an estimate for the growth of the solution in terms of an associated scalar ODE. Finally these results are applied to singular perturbations of second order Cauchy problems.

1. Introduction. We are concerned with the second order Cauchy problem

$$(\mathrm{ACP}_\varepsilon) \qquad \begin{aligned} &\varepsilon u_\varepsilon''(t) + 2Bu_\varepsilon'(t) = Au_\varepsilon(t), \qquad t \ge 0, \\ &u_\varepsilon(0) = u_0 \in D(A), \quad u_\varepsilon'(0) = u_1 \in D(A) \end{aligned}$$

in a Banach space E where A is the generator of a strongly continuous cosine family $(C_A(t))$ commuting with the bounded operator $B \in \mathscr{L}(E)$. It is well known that for $\varepsilon > 0$ $(\mathrm{ACP}_\varepsilon)$ is well-posed, i.e., it admits a unique solution which depends continuously on the initial conditions u_0 and u_1.

This paper is organized as follows. We first give (in case $\varepsilon = 1$) an explicit representation of the solution $u(\cdot)$ of (ACP_1) in terms of $C_A(t)$ and B. Then we use this formula to derive an estimate for the growth of $u(t)$. In fact, we associate with (ACP_1) a scalar ODE and show that its solution dominates $\|u(t)\|$. Finally these results are used to show convergence of $u_\varepsilon(\cdot)$ as $\varepsilon \downarrow 0$ to the unique solution of

$$(\mathrm{ACP}_0) \qquad \begin{aligned} &2Bu_0'(t) = Au_0(t), \qquad t \ge 0, \\ &u_0(0) = u_0 \end{aligned}$$

provided that the spectral bound of $-B$ is less than zero. Moreover, from the proof of this result we conclude that under the above assumptions AB^{-1} generates an analytic semigroup.

2. The explicit formula. In order to state the main result of this section we need the following definitions. For a bounded operator $Q \in \mathscr{L}(E)$ we define the *modified Bessel function of order zero* by

$$I_0(Q) := \sum_{n=0}^{\infty} \frac{(\frac{Q}{2})^{2n}}{(n!)^2}.$$