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ON MEANS OF DISTANCES ON THE SURFACE OF A SPHERE. II (UPPER BOUNDS)

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Given N points $x_1, \ldots, x_N$ on the unit sphere S in Euclidean d space $(d \geq 3)$, lower bounds for the deviation of the sum $\sum |x-x_j|^\alpha$, $\alpha > 1-d$; $x \in S$, from its mean value were established in terms of L^1-norms in the first part of this paper. In the present part it is shown that these bounds are best possible. Our main tool is a multidimensional quadrature formula with equal weights.

1. Introduction. On the surface $S = S^{d-1}$ of the unit sphere in d-dimensional Euclidean space E^d $(d \geq 3)$, we consider a certain class of distance functions and distance functionals, associated with a given N point set $\omega_N = \{x_1, x_2, \ldots, x_N\}$ on S. Denote by $|x-y|$ the Euclidean distance between two points x and y in E^d. Let $x \in S^{d-1}$ be a variable point. For each value of a parameter α $(1-d < \alpha < \infty)$ consider the distance function $U_\alpha(x, \omega_N)$ which we define as follows:

$$U_\alpha(x, \omega_N) = \sum_{j=1}^{N} |x-x_j|^\alpha - N \cdot m(\alpha, d) \quad \text{for } \alpha \neq 0,$$

and

$$U_0(x, \omega_N) = \sum_{j=1}^{N} \log|x-x_j| - N \cdot m(0, d).$$

Here $m(\alpha, d)$ denotes the mean value of $|x-x_j|^\alpha$ on S^{d-1}, i.e.

$$m(\alpha, d) = \frac{1}{\sigma(S)} \int_S |x-x_j|^\alpha \, d\sigma(x) \quad \text{for } \alpha \neq 0,$$
$$m(0, d) = \frac{1}{\sigma(S)} \int_S \log|x-x_j| \, d\sigma(x),$$

where σ is the $(d-1)$-dimensional area measure on S^{d-1}.

In the first part [**4**] we proved certain lower bounds for the L^1-norms of the functions $U_\alpha(x, \omega_N)$ (see Theorem 1 in [**4**]). The existence of such lower bounds is due to the fact that uniform distribution on S^{d-1}

[1] The author died on March 10, 1990 in a skiing accident in Austria.