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FOUR MANIFOLD TOPOLOGY AND GROUPS OF POLYNOMIAL GROWTH

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In this paper we present a new proof that groups of polynomial growth are "good" in the sense of Freedman and Quinn. It follows from the results of Freedman that the five dimensional s-cobordism theorem and the surgery exact sequence in dimension four hold for $\pi_1(M)$ of polynomial growth. In the processes, we will give a slightly more efficient procedure for grope height raising and a slightly different procedure for using the grope height to kill fundamental group images.

Introduction. The recent advances in four manifold topology can be loosely described as the result of new techniques for finding locally flat topological imbeddings of discs in four manifolds. Basically, Freedman has shown that a regular neighborhood of a certain finite construct, called a 1-story capped tower (with grope height at least four), contains a locally flat topologically imbedded disc (see **[3]** or **[5]**). The main difficulty in finding a capped tower in a 4-manifold M comes from the fundamental group $\pi_1(M)$. One says a group G is "good" (defined precisely below) if these difficulties can be overcome for $\pi_1(M) = G$ (and hence the 5-dimensional s-cobordism theorem and the surgery sequence in dimension 4 hold for $\pi_1 = G$). A group G is known to be "good" if it has polynomial growth (and more generally, if G is an elementary group).

M. Freedman has given two proofs that groups of polynomial growth are "good", both of which rely on nontrivial results from group theory. In the first proof **[3]** or **[5]**, one first shows that finite groups and the integers $\mathbf{Z}$ are "good". One then notes that the class of "good" groups is closed under the four operations: taking subgroups, forming quotient groups, taking direct limits and forming extensions. Therefore all elementary groups are "good". One then applies the result of **[6]** that groups of polynomial growth are almost nilpotent and hence elementary. In the second proof **[4]**, one shows by a direct argument that if the growth function ρ of a finitely generated group G satisfies $cr^d \leq \rho(r) \leq Cr^d$ for $r > 0$ and some (positive) constants c, C and d then G is "good". One then appeals to the result of **[6]** and **[1]** to