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SINGULAR MODULI SPACES OF STABLE VECTOR BUNDLES ON $\mathbf{P}^3$

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The goal of this paper is to give an example of singular moduli space of rank 3 stable vector bundles on $\mathbf{P}^3$.

Introduction.

In 1977/78, M. Maruyama proved the existence of a moduli scheme $M_{\mathbf{P}^n}(r; c_1, ..., c_{\min(r,n)})$ parametrizing isomorphic classes of rank r stable vector bundles on $\mathbf{P}^n$ with given Chern classes $c_1,..., c_{\min(n,r)}$ (cf. [**M1, M2**]). The goal of this note is to give, to the best of my knowledge, the first example of singular moduli space of stable vector bundles on $\mathbf{P}^3$. It has been motivated by a recent work of Ancona and Ottaviani where they show that the moduli space $MI_{\mathbf{P}^5}(k)$ of stable instanton bundles on $\mathbf{P}^5$ with quantum number k=3 or 4 is singular. Moreover they claim that $MI_{\mathbf{P}^5}(3)$ and $MI_{\mathbf{P}^5}(4)$ are the first examples of singular moduli spaces of stable vector bundles on projective spaces (cf. [**AO**]). Ancona-Ottaviani's result together with the well known fact that $M_{\mathbf{P}^2}(r; c_1, c_2)$ is a smooth quasi-projective variety of dimension $2rc_2 - (r-1)c_1^2 + 1 - r^2$ gives rise the following question:

Is there any example of singular moduli space of stable vector bundles on $\mathbf{P}^3$?

As I pointed out before my aim is to give an affirmative answer to this question (cf. Theorem 2.10).

1. Preliminaries.

In this section we recall some well known results needed later on.

1.1. Let H(18, 39) be the open subscheme of $Hilb\mathbf{P}_k^3$ parametrizing smooth connected curves $C \subset \mathbf{P}^3$ of degree 18 and genus 39. (See [**EF**] for a precise description of H(18, 39).) Let $H_1 \subset H(18,39)$ be the 72-dimensional irreducible, generically smooth component whose general point parametrizes an arithmetically Cohen-Macaulay curve $X \subset \mathbf{P}^3$ having a locally free resolution of the following type:

$$0 \to \mathcal{O}(-7)^4 \to \mathcal{O}(-6)^4 \oplus \mathcal{O}(-4) \to I_X \to 0. \tag{1}$$