PACIFIC JOURNAL OF MATHEMATICS
Vol. 173, No. 1, 1996

MINIMAL HYPERSPHERES IN TWO-POINT HOMOGENEOUS SPACES

PER TOMTER

In Riemannian geometry the study of minimal submanifolds has given the most important, higher-dimensional generalizations of geodesics. Especially significant from a global point of view are closed minimal submanifolds (generalizing closed geodesics); these raise many hard problems. In this paper we study existence and uniqueness questions in the case of the simplest topological type; i.e. minimal hyperspheres. We restrict ourselves to study such questions for the compact two-point homogeneous spaces; these spaces constitute the most natural generalization of classical (three-point homogeneous) spherical geometry. They can be characterized equivalently as (i) compact two-point homogeneous spaces, (ii) compact rank 1 symmetric spaces, or (iii) irreducible compact positively curved symmetric spaces. Since the standard spheres have been investigated in great detail in connection with the "Spherical Bernstein Problem", we only consider the complex projective spaces $\mathbf{C}P(n)$, the quaternionic projective spaces $\mathbf{H}P(n)$, and the Cayley projective plane $Ca(2)$ here.

Introduction.

The compact two-point homogeneous spaces are also precisely the symmetric spaces which have a homogeneous, minimal hypersphere, unique up to congruence, and determined as the principal isotropy group orbit of maximal volume. We call it the "equator" of the space. Hence this is the natural class of spaces to study uniqueness questions analogous to the Bernstein problem; i.e. the following question: Is any minimal hypersphere in a compact two-point homogeneous space an equator?

The most effective method has been the "equivariant differential geometry" initiated by Wu-Yi Hsiang and B. Lawson. In the case of a cohomogeneity two subgroup G of the isometry group the reduced minimal equation in the orbit space is the geodesic equation for a suitably chosen metric; and the problem of finding G-invariant closed minimal submanifolds is reduced to finding geodesics that may "close up" by satisfying a delicate intersection requirement with the singular boundary. The technical difficulties in proving