PACIFIC JOURNAL OF MATHEMATICS
Vol. 173, No. 1, 1996

(A_2)-CONDITIONS AND CARLESON INEQUALITIES IN BERGMAN SPACES

TAKAHIKO NAKAZI AND MASAHIRO YAMADA

Let ν and μ be finite positive measures on the open unit disk D. We say that ν and μ satisfy the (ν, μ)-Carleson inequality, if there is a constant $C > 0$ such that

$$\int_D |f|^2 d\,\nu \leq C \int_D |f|^2 d\,\mu$$

for all analytic polynomials f. In this paper, we study the necessary and sufficient condition for the (ν, μ)-Carleson inequality. We establish it when ν or μ is an absolutely continuous measure with respect to the Lebesgue area mesure which satisfy the (A_2)-condition. Moreover, many concrete examples of such measures are given.

§1. Introduction.

Let D denote the open unit disk in the complex plane. For $1 \leq p \leq \infty$, let L^p denote the Lebesgue space on D with respect to the normalized Lebesgue area measure m, and $\|\cdot\|_p$ represents the usual L^p-norm. For $1 \leq p < \infty$, let L^p_a be the collection of analytic functions f on D such that $\|f\|_p$ is finite, which are so called the Bergman spaces. For any z in D, let ϕ_z be the Möbius function on D, that is

$$\phi_z(w) = \frac{z-w}{1-\bar{z}w} \qquad (w \in D),$$

and put,

$$\beta(z, w) = 1/2\ \log(1 + |\phi_z(w)|)(1 - |\phi_z(w)|)^{-1} \quad (z, w \in D).$$

For $0 < r < \infty$ and z in D, set

$$D_r(z) = \{w \in D; \beta(z, w) < r\}$$

be the Bergman disk with "center" z and "radius" r, and we define an average of a finite positive measure μ on $D_r(a)$ by

$$\hat{\mu}_r(a) = \frac{1}{m(D_r(a))} \int_{D_r(a)} d\,\mu \qquad (a \in D),$$