

MIXED AUTOMORPHIC VECTOR BUNDLES ON SHIMURA VARIETIES

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Let $S^0(G, X)$, $S^0(G', X')$ be connected Shimura varieties associated to semisimple algebraic groups G , G' defined over $\mathbb{Q}$ and Hermitian symmetric domains X , X' . Let $\rho : G \rightarrow G'$ be a homomorphism of algebraic groups over $\mathbb{Q}$ that induces a holomorphic map $\omega : X \rightarrow X'$ mapping special points of X to special points of X' . Given equivariant vector bundles $\mathcal{J}$, $\mathcal{J}'$ on the compact duals $\check{X}$, $\check{X}'$ of the symmetric domains X , X' , we can construct a mixed automorphic vector bundle $\mathcal{M}(\mathcal{J}, \mathcal{J}', \rho)$, on $S^0(G, X)$ whose sections can be interpreted as mixed automorphic forms. We prove that the space of sections of a certain mixed automorphic vector bundles is isomorphic to the space of holomorphic forms of the highest degree on the fiber product of a finite number of Kuga fiber varieties. We also prove that for each automorphism τ of $\mathbb{C}$ the conjugate $\tau\mathcal{M}(\mathcal{J}, \mathcal{J}', \rho)$ of a mixed automorphic vector bundle $\mathcal{M}(\mathcal{J}, \mathcal{J}', \rho)$ on a connected Shimura variety $S^0(G, X)$ can be canonically realized as a mixed automorphic vector bundle $\mathcal{M}(\mathcal{J}_1, \mathcal{J}'_1, \rho_1)$ on another connected Shimura variety $S^0(G_1, X_1)$ associated to a semisimple algebraic group G_1 and a Hermitian symmetric domain X_1 .

1. Introduction.

Mixed automorphic forms generalize automorphic forms, and certain types of mixed automorphic forms occur naturally as holomorphic differential forms of the highest degree on certain fiber varieties over arithmetic varieties whose fibers are abelian varieties (see e.g. [8], [14], [16] [17], [18], [19]). Holomorphic automorphic forms can be interpreted as the sections of automorphic vector bundles on a Shimura variety (see [6], [7], [21], [22]) just as automorphic functions can be considered as sections of the sheaf of germs of functions on a Shimura variety. In this paper, we introduce mixed automorphic vector bundles on connected Shimura varieties whose sections can be interpreted as mixed automorphic forms.

Let E be an elliptic surface and let $\pi : E \rightarrow X$ be an elliptic fibration in the sense of Kodaira (cf. [11]). Thus E is a compact smooth surface over $\mathbb{C}$, X