COHOMOLOGY COMPLEX PROJECTIVE SPACE WITH DEGREE ONE CODIMENSION-TWO FIXED SUBMANIFOLDS

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If M^{2n} is a cohomology $\mathbb{C}P^n$ and p is a prime, let $D_p(M^{2n})$ be the set of positive integers d such that $d \in D_p(M^{2n})$ if there exists a diffeomorphism of M^{2n} of order p fixing an orientable, codimension-2 submanifold of degree d. If p = 2 or n is odd, then $1 \in D_p(M^{2n})$ implies that $D_p(M^{2n}) = \{1\}$. The case p odd and n even is also investigated. If M^{4m} is a homotopy $\mathbb{C}P^{2m}$ and $m \neq 0, 4$, or 7 (mod 8), then $1 \in D_3(M^{4m})$ implies that $D_3(M^{4m}) = \{1\}$.

1. Introduction.

A cohomology complex projective *n*-space is a smooth, closed, orientable 2n-manifold M^{2n} such that there is a class $x \in H^2(M; \mathbb{Z})$ with the property that $H^*(M; \mathbb{Z}) = \mathbb{Z}[x]/(x^{n+1})$. If $i: K^{2n-2} \subset M^{2n}$ is the inclusion map of a closed, connected, orientable submanifold and d is an integer, we will say that the degree of K^{2n-2} is d if $i_*[K]$ is the Poincaré dual of dx. We will always assume that the orientation of K^{2n-2} is chosen in such a way that d is nonnegative. Let p be a prime number and let G_p denote the cyclic group of order p. Let $D_p(M^{2n})$ be the set of positive integers d defined by the condition that $d \in D_p(M^{2n})$ if M^{2n} admits a smooth G_p action such that the fixed point set of the action contains a codimension-2 submanifold of degree d. If $d \in D_p(M^{2n})$, then $d \not\equiv 0 \pmod{p}$, (see [2, pp. 378-383]). The following conjecture is motivated by the work of several authors ([3], [4], [6], [8]).

Conjecture 1.0. If $D_p(M^{2n})$ is nonempty, then $D_p(M^{2n}) = \{1\}$.

This conjecture has been verified for small values of n ([3], Theorem A $(n = 3, p \ge 3, n = 4, p > 3)$, [4], Corollary 4.5 (n = 4, p = 2), [7], Theorem 1.7 (n = 4, p = 3)). We begin this paper with the observation that a weaker version of the conjecture is true if p = 2 or n is odd.

Theorem 1.1. Let M^{2n} be a cohomology complex projective n-space. (1) If p = 2 and $1 \in D_2(M^{2n})$, then $D_2(M^{2n}) = \{1\}$.