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RATIONAL PONTRYAGIN CLASSES, LOCAL REPRESENTATIONS, AND K^G-THEORY

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Suppose that X and Y are connected, simply connected Spin^c-manifolds of the same dimension. Let G be a compact connected Lie group with torsion-free fundamental group which acts upon X and Y such that X^G and Y^G are non-empty and consist entirely of isolated fixed points. Suppose that $f : X \to Y$ is a smooth G-map such that the induced map

$$f^* : K_G^*(Y) \to K_G^*(X)$$

is an isomorphism. If X and Y are even-dimensional then for each fixed point $x \in X^G$, the local representations of G at x and at $f(x)$ are equivalent. If $f : X \to Y$ is an equivalence then

$$f^* : H^*(Y; \mathbb{Q}) \to H^*(X; \mathbb{Q})$$

preserves Pontryagin classes.

1. Introduction.

Suppose that X and Y are compact smooth manifolds and $f : X \to Y$ is a smooth (homotopy) equivalence. In general, the map f does not preserve rational Pontryagin classes, which depend a priori upon the smooth structures on X and Y, unless f happens to be a diffeomorphism. S. P. Novikov proved in 1965 [**N1**, **N2**] that if f is a *homeomorphism* then rational Pontryagin classes are indeed preserved, and this remains the best general positive result on the subject. In 1981 Sullivan and Teleman jointly provided a proof of Novikov's result using differential geometric and analytic techniques, and recently Shmuel Weinberger gave a "short and conceptually simple analytic proof" of Novikov's theorem drawing upon new ideas in index theory for non-compact complete Riemannian manifolds. Baum and Connes [**BC**] have studied the foliated version of the problem and have positive results in the presence of negatively curved leaves.

In the early 1970's Ted Petrie developed a connection between the problem of preservation of Pontryagin classes and another classical problem. If G is a compact Lie group, X and Y are smooth G-manifolds, and $f : X \to Y$