PACIFIC JOURNAL OF MATHEMATICS
Vol. 175, No. 2, 1996

RAMANUJAN'S MASTER THEOREM FOR SYMMETRIC CONES

HONGMING DING, KENNETH I. GROSS AND DONALD ST. P. RICHARDS

Dedicated to R. A. Kunze, on the occasion of his sixty-fifth birthday

The Master Theorem of Ramanujan (1913), so named because of its centrality in much of Ramanujan's work on definite integrals, hypergeometric functions, and series expansions, relates coefficients in the Taylor's expansion of a function to the Mellin transform of the function over the interval $(0, \infty)$. In this paper we extend the setting of this classical theorem to apply to spherical series and spherical transforms on symmetric cones (also known as domains of positivity). To illustrate the range of applications of this theorem we obtain higher dimensional analogues of Carlson's uniqueness theorem for holomorphic functions, Newton's interpolation formula, and Mellin-Barnes integrals for certain hypergeometric functions.

Introduction.

Srinivasa Ramanujan Aiyangar, otherwise known as Ramanujan, needs no introduction, either to professional mathematicians or mathematical historians. He was born in poverty in India in 1887 and died not far from his place of birth at age 32. He was a self-taught mathematical genius possessing exceptional mathematical powers and a special originality and insight that defies comparison. Upon his death Ramanujan left a mountain of unpublished work, a great amount of which was contained in three Notebooks, together with three Quarterly Reports that were written in 1913 and communicated to the Board of Studies at University of Madras, where he was supported by a small research scholarship. Remarkably, only in the last decade, with the appearance of the wonderful book [1] by Berndt, has the material contained in these notebooks and reports been organized, analyzed, and published.

The subject of this paper is the generalization to symmetric cones (also known as domains of positivity) of Ramanujan's fundamental discovery described in his first quarterly report. Because this result was a touchstone for Ramanujan throughout his work on definite integrals, hypergeometric functions, and series expansions, it is known as *Ramanujan's Master Theorem.*