PACIFIC JOURNAL OF MATHEMATICS
Vol. 176, No. 2, 1996

A UNIQUENESS THEOREM FOR THE MINIMAL SURFACE EQUATION

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In 1991, Collin and Krust proved that if u satisfies the minimal surface equation in a strip with linear Dirichlet data on two sides, then u must be a helicoid. In this paper, we give a simpler proof of this result and generalize it.

1. Introduction.

Let $\Omega_\alpha \subset \mathbb{R}^2$ be a sector domain with angle $0 < \alpha < \pi$. Consider the minimal surface equation

$$\operatorname{div} Tu = 0 \tag{1}$$

where $Tu = \dfrac{\nabla u}{\sqrt{1+|\nabla u|^2}}$ and ∇u is the gradient of u. In 1965, Nitsche [7] announced the following results:

(1) Given a continuous function f on $\partial\Omega_\alpha$, there always exists a solution u which satisfies the minimal surface equation in Ω_α with Dirichlet data f on $\partial\Omega_\alpha$;

(2) If u satisfies the minimal surface equation with vanishing boundary value in Ω_α, then $u \equiv 0$.

Nitsche thus raised the following question: Let $\Omega \subset \Omega_\alpha$ and let f be an arbitrary continuous function on $\partial\Omega$. If the Dirichlet problem

$$\begin{cases} \operatorname{div} Tu = 0 & \text{in } \Omega, \\ u = f & \text{on } \partial\Omega \end{cases}$$

has a solution, is it unique?

We notice that similar questions for higher dimensions are raised in [6]. Results in this direction were obtained by Miklyukov [5] and Hwang [4] independently, in which the following result was established:

Theorem 1. *Let $\Omega \subset \mathbb{R}^2$ be an unbounded domain and let $u, v \in C^2(\Omega) \cap C^0(\overline{\Omega})$. For every $R > 0$, set $B_R = \{x \in \mathbb{R}^2 \mid |x| < R\}$ and $\Gamma_R = \partial(\Omega \cap B_R) \cap$*