PACIFIC JOURNAL OF MATHEMATICS
Vol. 30, No. 3, 1969

THE 2-CELL AS A PARTIALLY ORDERED SPACE

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In this paper we prove a Jordan Curve Theorem (Theorem 1) for certain two dimensional partially ordered spaces.

We use this result to give a new characterization of the closed 2-cell (Theorm 2).

By a *partially ordered space* we X mean a Hausdorff space X with a partial order which is closed when regarded as a subset of $X \times X$ ($X \times X$ has the product topology).

For $x \in X$ we set

$$L(x) = \{y \in X \mid y \leqq x\}$$
$$M(x) = \{y \in X \mid x \leqq y\}$$

and

$$\Gamma(x) = L(x) \cup M(x) .$$

If $A \subset X$ we let

$$L(A) = \bigcup \{L(x) \mid x \in A\} .$$

We define $M(A)$ and $\Gamma(A)$ analogously. We let L (resp. M) denote the set of minimal (resp. maximal) elements of X.

A *chain* is a totally ordered set. An *order arc* is a compact and connected chain. A separable and nondegenerate order arc is homeomorphic to $[0, 1]$. A *continuum* is a compact, connected, Hausdorff space. An *arc* is a continuum with exactly two noncutpoints. A *circle* is a continuum such that every pair of points separates it.

DEFINITION. If X is a partially ordered space and $A \subset X$ let

$$C(A) = L(A) \cap M(A) .$$

A subset A of X is *convex* if and only if $A = C(A)$.

L. Nachbin proved the following result ([4], p. 48).

LEMMA 1.1. (*Nachbin*). *A compact partially ordered space X has a basis of convex open sets.*

The following three lemmas appear in [5]. For completeness we sketch their proofs here.

LEMMA 1.2. *Let X be a compact partially ordered space such*