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STABILITY THEOREMS FOR LIE ALGEBRAS OF DERIVATIONS

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Let A be a finite dimensional algebra over a field F of characteristic zero and let L be a completely reducible Lie algebra of derivations of A. If A is associative, then there exists an L-invariant Wedderburn factor of A. If A is a Lie algebra, there exists an L-invariant Levi factor of A. If A is a solvable Lie algebra, there exists an L-invariant Cartan subalgebra of A. This paper deals with the uniqueness of such L-invariant subalgebras. For the associative case the assumption of characteristic zero can be dropped if we assume that the radical of A is L-invariant.

2. Preliminaries. If A is a finite dimensional associative algebra over a field F with radical R such that A/R is separable (that is, semisimple and remains so under every field extension of F), then the Wedderburn principal theorem states that there exists a separable subalgebra S such that $A = S + R, S \cap R = \{0\}$. S is called a Wedderburn factor of A. Since R is nilpotent, for r in R, $(1 - r)^{-1} = 1 + r + \cdots + r^{n-1}$, where $r^n = 0$. Let C_{1-r} be the inner automorphism of A defined by conjugation by the invertible element $1 - r$. The Malcev Theorem states that if S is any separable subalgebra of A and T is a Wedderburn factor of A, then there exists r in R such that $C_{1-r}(S) \subseteqq T$. Thus, the Wedderburn factors of A are just the maximal separable subalgebras. See [4] for the above information. In § 3 it is shown that if L is completely reducible (every L-invariant subspace of A has a complementary L-invariant subspace), F arbitrary, R L-invariant, and S, T two L-invariant Wedderburn factors of A, then there exists an element r in R such that $C_{1-r}(S) = T$ and $D(r) = 0$ for all D in L. Such an element r is called an L-constant.

If A is a Lie algebra over a field F of characteristic zero and R is the radical (maximal solvable ideal) of A, then the Levi theorem states that $A = S + R$, $S \cap R = \{0\}$, where S is a semisimple subalgebra of A isomorphic to A/R. S is called a Levi factor of A. The Malcev-Hanish-Chandra theorem states that any two Levi factors of A are conjugate by an automorphism $\exp(Adx)$, where x is in N, the nil radical (maximal nilpotent ideal) of A. In § 4 it is shown that for L completely reducible and S, T L-invariant Levi factors of A, then there is an L-constant x in N such that $\exp(Adx)(S) = T$.

If A is a solvable Lie algebra over a field F of characteristic zero, then any two Cartan subalgebras are conjugate by an automorphism