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# GENERALIZED BOL FUNCTIONAL EQUATION

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**Each identity in a group or in a quasigroup induces a generalized identity (functional equation) in a class of quasigroups. Generalized associativity, generalized bisymmetry and generalized distributivity are examples of such generalized identities. From the left Bol identity**

$$x(y(xz)) = (x(yx))z$$

**on a quasigroup, we obtain a generalized Bol identity on a class of quasigroups:**

$$A_1(x, A_2(y, A_3(x, z))) = A_4(A_5(x, A_6(y, x)), z)\ ,$$

**where the $A_i$'s are quasigroup operations on a set $Q$. The general solution of this generalized Bol functional equation is obtained by reducing it to another functional equation**

$$P(x, y + S(x, z)) = P(x, y + \alpha(x)) + z$$

**where $P$ and $S$ are quasigroup operations on $Q$ and $\alpha(x) = S(x, 0)$. If the operations in the last functional equation are considered on real numbers (or groups), then the solution of this equation is obtained.**

One of the most important identities considered in the theory of quasigroups is Bol identity. A loop $Q(\cdot)$ isc alled a left Bol loop [2] if the following identity

$$x(y(xz)) = (x(yx))z\ , \tag{1}$$

holds for every $x, y, z \in Q$. The identity (1) is called the left Bol identity. The right Bol identity is defined analogously

$$((zx)y)x = z((xy)x)\ . \tag{2}$$

For more information of algebraic properties of Bol loops, see for example [4]. If a loop is both a right and a left Bol, then it is a Moufang loop, i.e. one of the following Moufang identities are satisfied:

$$x(y(xz)) = ((xy)x)z\ , \tag{3}$$

$$((zx)y)x = z(x(yx))\ . \tag{4}$$

It is easily seen that (3) is a particular case of (2); if $Q(\cdot)$ satisfies the elasticity law $(xy)x = x(yx)$, then (1) implies (3). On the other hand the left Moufang identity (3) does not imply (1), see for example [3].