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SOBOLEV APPROXIMATION BY A SUM OF SUBALGEBRAS ON THE CIRCLE

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Let ψ be an orientation-reversing homeomorphism of the unit circle onto itself. We consider approximation in certain Sobolev norms by functions of the form $f(z) + g(\psi)$, where f and g are polynomials. The methods involve conformal welding and Hardy space theory. We construct a Jordan arc of positive continuous analytic capacity such that the harmonic measures for the two complementary domains are mutually absolutely continuous.

Let $C(S)$ denote the space of continuous, complex-valued functions on the unit circle S. For $f \in C(S)$, let $P(f)$ denote the space of all polynominals in f with complex coefficients. Let z denote the identity function. Browder and Wermer proved the following theorem [3, p. 551].

> *Let ψ be a direction-reversing homeomorphism of S onto S. Then the vector space sum $P(z) + P(\psi)$ is uniformly dense in $C(S)$.*

In the first part of this paper we prove a result that partially extends this theorem to the C^1 norm. We say that a direction-reversing homeomorphism $\psi: S \to S$ is an *involution* if $\psi \circ \psi = z$. Let $C^1(S)$ denote the space of continuously-differentiable, complex-valued functions on S, with the norm

$$\|f\|_{C^1} = \|f\|_\infty + \left\|\frac{df}{d\theta}\right\|_\infty .$$

If $\alpha > 0$, let $C^{1+\alpha}(S)$ denote the space of functions f in $C^1(S)$ such that $f' = df/d\theta$ satisfies a Lipschitz condition with exponent α:

$$|f'(a) - f'(b)| \leqq K|a - b|^\alpha$$

for all points a and b in S.

THEOREM. *Let $\alpha > 0$, and let $\psi \in C^{1+\alpha}(S)$ be an involution. Then $P(z) + P(\psi)$ is dense in $C^1(S)$.*

Our proof of this theorem is radically different from Browder and Wermer's proof of their result. By means of duality and conformal welding, we reduce the theorem to a statement about