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ON THE APPROXIMATION OF SINGULARITY SETS BY ANALYTIC VARIETIES

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We study the problem of approximating the singularity set of an analytic function of two complex variables, lying in a product domain in C^2, by analytic varieties.

Let D denote the open unit disk and let

$$D \times \mathbf{C} = \{(z, w) \mid z \in D, w \in \mathbf{C}\} \subset \mathbf{C}^2.$$

We consider a compact set X contained in the unit bidisk $|z| \leq 1$, $|w| \leq 1$. Let X^0 denote $X \cap (D \times \mathbf{C})$. We assume that there exists a function ϕ which is analytic on $D \times \mathbf{C} \setminus X^0$ and singular at each point of X^0. If there exists such a ϕ we call X a *singularity set*.

For each λ in $\bar{D}$ we put

$$X_\lambda = \{w \in \mathbf{C} \mid (\lambda, w) \in X\}.$$

Each X_λ is then a compact subset of $|w| \leq 1$. We assume $X_\lambda \neq \varnothing$, for each λ.

Singularity sets were first studied by Hartogs, in [3]. Hartogs showed that if for some integer p X_λ contains at most p points for each λ, then X^0 is an analytic subvariety of $D \times \mathbf{C}$. Further results on singularity sets were given by Oka, [5], and Nishino, [4].

Recently one of us in [7] and Slodkowski in [6] studied general singularity sets. In particular, Theorem 1 in [7] gives that the maximum principle holds on X^0 for restrictions to X^0 of polynomials in z and w, in the sense that for each compact subset N of X^0 and each $(z_0, w_0) \in N$,

$$|P(z_0, w_0)| \leq \max_{\partial N} |P|,$$

for each polynomial P.

(See also [6], Theorem II, (vi).) Here ∂N denotes the boundary of N relative to X^0. In particular, fix $R < 1$. Put $N = \{(\lambda, w) \in X \mid |\lambda| \leq R\}$. Then $\partial N = \{(\lambda, w) \in X \mid |\lambda| = R\}$. Hence, for $(z_0, w_0) \in X$, $|z_0| < R$, we have for each polynomial P,

$$|P(z_0, w_0)| \leq \max_{X \cap (|z| = R)} |P|.$$