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RADON-NIKODYM PROBLEM FOR THE VARIATION OF A VECTOR MEASURE

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We consider the problem of representing the variation $|m|$ of a vector measure m as an integral in the Dinculeanu sense with respect to M.

Throughout this paper (S, Σ) denotes a measurable space. If X is a Banach space, we write X^* for the dual space and K_X for the closed unit ball of X. We use brackets $\langle\ ,\ \rangle$ for the pairing between a Banach space and its dual. Let $m\colon \Sigma \to X$ be a vector measure with finite variation $|m|$. Recall that a strongly measurable function $f\colon S \to X^*$ is said to be integrable in Dinculeanu's sense if there exists a sequence $\{f_n\}_{n\geq 1}$ of simple functions converging $|m|$-a.e. to f such that

$$\lim_{n,p\to\infty} \int \|f_n - f_p\|\, d|m| = 0,$$

i.e., the function $\|f\|$ is $|m|$-integrable. Further, D-$\int_A f\,dm$ denotes the Dinculeanu integral of the function f with respect to m over the set A.

It was proved in [2] that for every $\varepsilon > 0$ there exists an X^*-valued strongly measurable function f defined on the set S such that $\|f\| \leq 1 + \varepsilon |m|$-a.e. and $|m|(A) =$ D-$\int_A f\,dm$ for each $A \in \Sigma$. We are interested in the following question: For which Banach spaces may we obtain the preceding equality when we insist that $\|f\| = 1$ a.e. $|m|$?

We begin our investigation by introducing the following property of Banach spaces. The Banach space X has property (DV) if for every equivalent norm on x, for every measurable space (S, Σ) for every equivalent norm on X and every vector measure $m\colon \Sigma \to X$ with finite variation $|m|$ there exists a strongly measurable function $f\colon S \to X^*$ with $\|f\| = 1$ $|m|$-a.e. such that $|m|(A) =$ D-$\int_A f\,dm$ for each $A \in \Sigma$.

THEOREM 1. *If both X and X^* have the Radon-Nikodym Property, then X has property* (DV).