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STABLE RELATIONS II: CORONA SEMIPROJECTIVITY AND DIMENSION-DROP C^*-ALGEBRAS

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We prove that the relations in any presentation of the dimension-drop interval are stable, meaning there is a perturbation of all approximate representations into exact representations. The dimension-drop interval is the algebra of all M_n-valued continuous function on the interval that are zero at one end-point and scalar at the other. This has applications to mod-p K-theory, lifting problems and classification problems in C^*-algebras. For many applications, the perturbation must respect precise functorial conditions. To make this possible, we develop a matricial version of Kasparov's technical theorem.

1. Introduction.

Suppose $\mathcal{R}$ is a finite set of relations on a finite set G of generators so that $C^*\langle G|\mathcal{R}\rangle$ is isomorphic to the dimension-drop interval

$$\tilde{\mathbb{I}}_n = \{f \in C[0,1] \mid f(0), f(1) \in \mathbb{C}I\}.$$

For simplicity, we assume the relations are of the form $p(g_1, \dots, g_n) = 0$ for some $*$-polynomial p. *Weak stability* means that an approximate representation $(x_1, \dots, x_n)$, meaning an n-tuple of elements in a C^*-algebra A such that each $p(x_1, \dots, x_n)$ is close zero, can be perturbed slightly within A to an actual representation $(\tilde{x}_1, \dots, \tilde{x}_n)$. That this (and a little more) can be done was shown in [**8**], but only for one specific set of relations. The relations $\mathcal{R}$ are *stable* if the pertubation can be done so that whenever there is a $*$-homomorphism $\varphi : A \to B$ which sends $(x_1, \dots, x_n)$ to an exact representation, then $\varphi(\tilde{x}_j) = \varphi(x_j)$.

There are several advantages to stability over weak stability. It is far more useful when dealing with extensions of C^*-algebras and it depends only on the universal C^*-algebra, not the choice of relations for that C^*-algebra. The reason for our focus on the dimension-drop interval is primarily that this is the most complicated building block used in the inductive limits, called AD algebras, that appeared in Elliott's first classification paper [**7**].