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PERIODIC SOLUTIONS OF A TWO-POINT BOUNDARY VALUE PROBLEM AT RESONANCE

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1. Introduction

We consider the periodic boundary value problem

$$
\begin{aligned}
&u'' + f(u)u' + g(x,u) = h \text{ in } (0,2\pi),\\
&u(0) - u(2\pi) = u'(0) - u'(2\pi) = 0,
\end{aligned} \tag{1.1}
$$

where $h \in L^1(0,2\pi)$ is given, $f : \mathbf{R} \to \mathbf{R}$ is a continuous function and $g : (0,2\pi) \times \mathbf{R} \to \mathbf{R}$ is a Caratheodory function. That is, $g(x,u)$ is continuous in $u \in \mathbf{R}$ for a.e. $x \in (0,2\pi)$, is measurable in $x \in (0,2\pi)$ for all $u \in \mathbf{R}$ and satisfies for each $r > 0$, there exists $a_r \in L^1(0,2\pi)$ such that

$$
|g(x,u)| \le a_r(x) \tag{1.2}
$$

for a.e. $x \in (0,2\pi)$ and all $|u| \le r$. Concerning the growth condition of the nonlinear term g, we assume that either

(H) There exist a constant $r_0 > 0$, and $a, b, c, d \in L^1(0,2\pi)$, $a, b \ge 0$ and $a(x) \le 1$ for a.e. $x \in (0,2\pi)$ with strict inequality on a positive measurable subset of $(0,2\pi)$, such that for a.e. $x \in (0,2\pi)$ and all $u \ge r_0$

$$
c(x) \le g(x,u) \le a(x)|u| + b(x),
$$

and for a.e. $x \in (0,2\pi)$ and all $u \le -r_0$

$$
-a(x)|u| - b(x) \le g(x,u) \le d(x);
$$

or

(G) There exist a constant $r_0 \ge 0$, and $a, b, c, d \in L^1(0,2\pi)$, $a, b \ge 0$ and $a(x) \le 1/4$ for a.e. $x \in (0,2\pi)$ with strict inequality on a positive measurable subset of $(0,2\pi)$, such that for a.e. $x \in (0,2\pi)$ and all $u \ge r_0$

$$
c(x) \le g(x,u),
$$

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