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ASYMPTOTIC BEHAVIOR OF RADIAL SOLUTIONS TO AN ELLIPTIC-PARABOLIC SYSTEM WITH NONLINEAR BOUNDARY CONDITIONS

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1. Introduction

Chemical reactions that take place in a bounded domain are often described by some reaction-diffusion systems with linear boundary conditions. Such kinds of reaction-diffution systems have been investigated by many reseachers, e.g., Rothe [16], Feng [4], Hoshino-Yamada [5], and others (see also Ruan [17, Theorem 5. 1]). On the other hand interfacial reactions, i.e., chemical reactions that take place on the interface between two phases (as oil and water), are often described by systems of diffusion equations with coupled, nonlinear boundary conditions. Also some important interfacial reactions in chemical engineering are described by elliptic-parabolic systems with coupled, nonlinear boundary conditions. Unfortunately it is difficult to deal with coupled, nonlinear boundary conditions by standard techniques. In fact, not so many fundamental theories are known concerning parabolic systems with non-monotonous, coupled, nonlinear boundary conditions. Recently, surmounting these difficulties, several mathematicians have investigated some systems of 1-dimensional diffusion equations with nonlinear boundary conditions that are related to interfacial reactions (see Yamada-Yotsutani [19], Shinomiya [18], Nagasawa [15], Iida-Yamada-Yotsutani [7], [8], [9], Iida-Yamada-Yanagida-Yotsutani [11], Iida-Ninomiya [6]; see also [17] and the references therein). As for elliptic-parabolic systems related to interfacial reactions, however, there seems to have been no investigations except Yotsutani [21], in which the existence and uniqueness of solutions are shown. The present paper is a first trial to construct a fundamental theory on asymptotic behavior of solutions to such an elliptic-parabolic system with coupled, nonlinear boundary conditions.

Let r_0, r_1 be given numbers with $0<r_0<r_1<1$, and put

$$\begin{aligned}
&\Omega_0=\{x\in \boldsymbol{R}^2;\ |x|<r_0\}, && \Gamma_0=\{x\in \boldsymbol{R}^2;\ |x|=r_0\},\\
&\Omega_*=\{x\in \boldsymbol{R}^2;\ r_0<|x|<r_1\}, && \Gamma_1=\{x\in \boldsymbol{R}^2;\ |x|=r_1\},\\
&\Omega_1=\{x\in \boldsymbol{R}^2;\ r_1<|x|<1\}, && \Gamma_2=\{x\in \boldsymbol{R}^2;\ |x|=1\},
\end{aligned}$$

$$\Omega=\Omega_0\cup\Omega_*\cup\Omega_1$$