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DISCRETE SPECTRUM OF SCHRÖDINGER OPERATORS WITH PERTURBED UNIFORM MAGNETIC FIELDS

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1. Introduction

In this paper we study a Schrödinger operator with a magnetic field:

$$H=(-i\nabla-b(x))^2+V(x) \tag{1.1}$$

defined on $C_0^\infty(\boldsymbol{R}^3)$, where $V\in L^2_{loc}(\boldsymbol{R}^3)$ is a scalar potential and $b\in C^1(\boldsymbol{R}^3)^3$ is a vector potential, both of which are real-valued, and $\vec{B}(x)=\nabla\times b$ is called the magnetic field. Let $x=(x_1, x_2, z)\in\boldsymbol{R}^3$, $\vec{\rho}=(x_1, x_2)$, $r=|x|$, $\rho=|\vec{\rho}|$, and $\nabla_2=(\partial/\partial x_1, \partial/\partial x_2)$. Letting $T=-i\nabla-b(x)$, we define the quadratic form q_H by

$$q_H[\phi, \psi]=\int_{\boldsymbol{R}^3}(T\phi\cdot\overline{T\psi}+V\phi\bar{\psi})dx,$$
$$q_H[\phi]=q_H[\phi, \phi]$$

for ϕ, $\psi\in C_0^\infty(\boldsymbol{R}^3)$. We assume that

$$V(x)\to 0 \text{ as } |x|\to\infty. \tag{V1}$$

Then H admits a unique self-adjoint realization in $L^2(\boldsymbol{R}^3)$ (denoted by the same notation H) with the domain

$$D(H)=\{u\in L^2(\boldsymbol{R}^3)\,;\,|V|^{1/2}u,\ Tu,\ Hu\in L^2(\boldsymbol{R}^3)\},$$

which is associated with the closure of q_H (denoted by the same notation q_H) with the form domain

$$Q(H)=\{u\in L^2(\boldsymbol{R}^3)\,;\,|V|^{1/2}u,\ Tu,\in L^2(\boldsymbol{R}^3)\},$$

This fact can be proved in the same way as in the cases of the constant magnetic fields ([1] and [7]).

It is well known that, if $\vec{B}(x)\equiv 0$, then the finiteness or the infiniteness of the discrete spectrum of H depends on the decay order of the scalar potential V, of which the border is $|x|^{-2}$([6]). On the other hand, if $\vec{B}(x)\equiv(0, 0, B)$, B being a positive constant, then the number of the discrete spectrum of H is infinite under