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A NOTE ON AUSLANDER–REITEN QUIVERS FOR INTEGRAL GROUP RINGS

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1. Introduction

Let G be a finite group and $\mathcal{O}$ be a complete discrete valuation ring, with the maximal ideal (π) and residue field $k=\mathcal{O}/(\pi)$ of characteristic $p>0$. R will be used to denote either $\mathcal{O}$ or k. Let Θ be a connected component of the stable Auslander-Reiten quiver $\Gamma_S(RG)$ of the group algebra RG and set $V(\Theta)=\{vx(M)|M$ is an indecomposable RG-module in $\Theta\}$, where $vx(M)$ denotes the vertex of M. Due to Kawata ([4, Proposition 3.2]), we know that there is a minimal element Q in $V(\Theta)$ with respect to the partial order $\leq_G$ which is uniquely determined up to G-conjugation. We call Q a vertex of Θ.

Let $N=N_G(Q)$ and f be the Green correspondence with respect to (G,Q,N). Choose an indecomposable RG-module M_0 in Θ with Q as its vertex. Let Δ be the connected component of $\Gamma_S(RN)$ containing $fM_0=L_0$. In the case $R=k$, Kawata has shown the following theorem, which extends the Green correspondence, in his paper [4]:

There is a graph monomorphism from Θ to Δ which preserves edge-multiplicity and direction.

The purpose of this note is to ensure that the above result also holds for $\mathcal{O}G$-lattices (i.e., finitely generated $\mathcal{O}$-free $\mathcal{O}G$-modules). The important tools used here can be found in [4], indeed the whole argument in [4] is also valid for $\mathcal{O}G$-lattices with some modifications. In this note, we shall provide a slightly simple proof by examining the middle terms of Auslander-Reiten sequences (see Theorem 2.5 and Corollary 2.6 below). Our approach is valid for both $\mathcal{O}G$ and kG, and will make it clearer that Kawata's graph morphism is an extension of the Green correspondence. The graph morphism stated above is not always isomorphic. In Section 3, we shall give an example of $\mathcal{O}G$-lattices such that the graph morphism is actually not isomorphism on the component containing them.

The notation is almost standard. We shall work over the group ring RG. All the modules considered here are finitely generated free over R. We write $W|W'$