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# THE AUSLANDER-REITEN QUIVER OF A RING OF RIGHT LOCAL TYPE

Dedicated to Professor Manabu Harada on his 60th birthday

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Throughout this paper, $A$ is a right and left artinian ring and $J$ is the Jacobson radical of $A$. All modules are assumed to be finitely generated over $A$. As well known Auslander-Reiten sequences (abbreviated AR-sequences) exist over arbitrary algebras and over representation-finite rings [6], [7], [27]. An AR-sequence of modules over a ring $A$ and the indecomposable decomposition of the middle term of this sequence define a mesh in the Auslander-Reiten quiver (AR-quiver for short) of $A$, and meshes determine the AR-quiver of $A$. For the representation-finite rings the computation of AR-quiver gives all indecomposable modules and all non-isomorphisms between them up to isomorphisms. In the algebra case we have a general way to compute AR-sequences stopping at any non-projective modules [11], [14] although the computation of the decomposition of the middle terms is not clear. However this computation is heavily depends on the existence of the selfduality of algebras. Hence for an arbitrary artinian ring even for a representation-finite ring, this computation is not available. Over such a ring we therefore have to compute AR-sequences individually.

In this paper we will compute (1) all the AR-sequences including the indecomposable decompositions of the middle terms (Theorem I), (2) full subquivers of the AR-quiver which give the whole of the AR-quiver by gluing together and (3) all the meshes (including values) in the AR-quiver (Theorem II), over a special type of representation-finite rings, namely over a ring of right local type. As a corollary we will obtain that the AR-quiver of a ring $A$ of right local type is a *well valued* translation quiver (i.e. the value $(a, b)$ of any arrow of the AR-quiver maps to $(b, a)$ by the translation between arrows associated to the AR-translation; see section 3 for detail) iff $A$ is, in addition, a ring of left colocal type. Further for a ring $A$ defined by a bimodule $M$ over division rings, we will give a necessary and sufficient condition for $A$ to be of right local type in terms of the dimension sequence of $M$.

In section 1 we first quote Sumioka's result on a ring of right local type,