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FREDHOLM DETERMINANT FOR PIECEWISE LINEAR TRANSFORMATIONS

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1. Introduction

Let F be a piecewise linear transformaion from a finite union of bounded intervals I into itself and P be the Perron-Frobenius operator assoicated with it. Since F is piecewise smooth, P can be expressed as

$$Pf(x) = \sum_{y:F(y)=x} f(y)|F'(y)|^{-1} \qquad \text{a.e.x}$$

for $f \in L^1$, the set of all Lebesgue integrable functions on I. In this paper, we assume that

$$(1) \qquad \xi = \operatorname*{essinf}_{x\in I} \liminf_{n\to\infty} n^{-1} \log |F^{n\prime}(x)| > 0 .$$

We call the number ξ the lower Lyapunov number. We will study Spec(F), the spectrum of $P|_{BV}$, the restriction of P to the subspace BV of functions with bounded variation. The generating function of P is determined by the orbits of the division points of the partition, and the orbits are characterized by a finite dimensional matrix $\Phi(z)$ which is defined by a renewal equation (§ 3). Hence, we can show that $D(z)=\det(I-\Phi(z))$, which we call a Fredholm determinant, is the determinant of $I-zP=\sum_{n=0}^{\infty} z^n P^n$ in the following sense:

Theorem A. *Let $\lambda \in C$ and assume that $|\lambda| > e^{-\xi}$. Then λ belongs to* Spec(F) *if and only if $z=\lambda^{-1}$ is a zero of $D(z)$:*

$$D(\lambda^{-1}) = 0 .$$

Furthermore, such λ is an eigenvalue of $P|_{BV}$.

We can calculate the eigenvalues of $P|_{BV}$ concretely by this theorem. Some examples are shown in § 6. Also we can prove the following intrinsic characterization of the power series $D(z)$ from a detailed re-examination of the renewal equation. Let us denote the following Ruelle-Artin-Mazur zeta function by $\zeta(z)$: