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# MODULI SPACES OF YANG-MILLS CONNECTIONS OVER QUATERNIONIC KÄHLER MANIFOLDS

Dedicated to Professor Shingo Murakami on his sixtieth birthday

TAKASHI NITTA

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## Introduction

The concept of anti-self-dual connections plays an important role in Yang-Mills theory for 4-manifolds (cf. Atiyah's monograph [1]). For instance, Atiyah, Hitchin and Singer [2] determined the moduli space of instantons on $S^4$ by differential geometric method, while Hartshorne [5] obtained the same result via twistor theory by showing that the moduli space of instantons over $S^4$ is the real part of the moduli space of null-correlation bundles over $P^3(\boldsymbol{C})$.

Now the purpose of this paper is to give a generalization of the result of Hartshorne [5] in the following way. We have the notion of $B_2$-connections $\nabla$ on vector bundles over quaternionic Kähler manifolds $M$ as higher dimensional analogue of anti-self-dual connections over 4-manifolds (cf. [3], [11], [15]). Let $p: Z \to M$ be the twistor space. Then, to each $B_2$-connection $\nabla$ over $M$, we can associate in a unique way an Einstein-Hermitian connection $\tilde{\nabla} := p^*\nabla$ over $Z$. Our main result is:

**Theorem.** *The mapping $\nabla \mapsto \tilde{\nabla}$ natually induces an embedding of the moduli space of $B_2$-connections over $M$ as a totally real submanifold of the moduli spcae of Einstein-Hermitian connections over $Z$.*

In a forthcoming paper, we shall give a compactification of the moduli space of Einstein-Hermitian connections for null-correlation bundles on $P^{2m+1}(\boldsymbol{C})$.

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## 1. Notation, conventions and preliminaries

For this section we refer to [6], [7], [8], [9], [10] and [11].

Let $N$ be a compact complex manifold and $(F, h_F)$ a Hermitian vector bundle over $N$ where $F$ is a $C^\infty$ complex vector bundle and $h_F$ is a Hermitian metric on $F$.