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SAMPLE PATH PROPERTIES OF ERGODIC SELF-SIMILAR PROCESSES

Dedicated to Professor Nobuyuki Ikeda on his 60th birthday

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1. Introduction

In this paper we shall study some sample path properties of self-similar processes with ergodic scaling transformations, in particular, a class of stable self-similar processes which includes the fractional stable processes. A large number of papers on sample path properties have been devoted to Gaussian processes and Lévy processes, i.e. stochastic processes with independent, stationary increments. In case of the Brownian motion, especially, we have Kolmogorov's test as a refinement of the law of the iterated logarithm and Chung-Erdös-Sirao's test (cf. [4]) as a refinement of Lévy's modulus of continuity.

We shall show some zero-one laws on sample path properties for general self-similar processes with ergodic scaling transformations in Sections 2 and 5. In Sections 3 and 4, we shall be concerned with a class of stable self-similar processes having stationary increments. We shall give integral tests for upper and lower functions with respect to the local growth of sample paths, which correspond to Kolmogorov's test and also to the results of Khinchin [15] for strictly stable processes. With respect to the uniform growth, in case of fractional stable processes with continuous sample paths, we shall give criteria for upper and lower functions. Furthermore, we shall show the existence of function which is neither an upper nor a lower function. This fact sharply contrasts with the Brownian motion case (cf. [4]).

Various sample path properties of self-similar processes with ergodic scaling transformations can be shown to hold with probability zero or one. Among such properties, we shall study growth properties in Section 2 and Hausdorff measure properties in Section 5. The results in Section 2 enable us to prove the above mentioned results in Section 3 by using an extension of Borel-Cantelli's lemma given in [16] rather than that of [3]. These zero-one laws on