Yoshida, M.
Osaka J. Math.
23 (1986), 415–423

ON DISORDER PROBLEM WITH POINT PROCESSES

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(Received March 15, 1985)

0. Introduction

In this paper the following special optimal stopping problem called "disorder problem" is considered: on some probability space $(\Omega, \mathcal{F}, P; \mathcal{F}_t)$ we are given an observable point process $(\xi_t)_{t\geq 0}$, and unobservable random variable Θ with values in $\boldsymbol{R}_+$. The stochastic characteristics of $(\xi_t)_{t\geq 0}$ may change at the random moment of time Θ, the probability law of Θ is known, however its value can not be observed directly. The objective is to maximize the value $E[g(\Theta, \tau)]$ by selecting a stopping time τ that is adapted to $\{\sigma(\xi_s, s\leqq t)\}_{t>0}$, for some given reward function $g(s, t)$. This kind of problems are considered in [1], [2], [3], [7], [8] and [10]

In section 2 according to the general theorem for optimal stopping problems with continuous parameter processes posed by M.E. Thompson [9], we derive the form of an optimal stopping time and the maximum expected reward function. In section 3 we restrict ourselves to the case when the expected reward process forms a monotone process and we apply the theorem of A. Irle [4] to our problem, and then we derive a form of optimal stopping time. At the end of section 3 we consider a special example and fined an optimal stopping time explicitly.

1. Statement of problem and preliminaries

Consider a measurable space $(\boldsymbol{X}, \mathcal{B})$ where $\boldsymbol{X}$ is a space of piecewise constant functions $x=(x_t)$, $t\geqq 0$, such that $x_0=0$ and $x_t=x_{t-}+(0 \text{ or } 1)$, $\mathcal{B}$ is a σ-algebra $\sigma\{x_s; s\geqq 0\}$. On $(\boldsymbol{X}, \mathcal{B})$ we are given complete probability measures μ^1 and μ^2, which satisfy Assumption I given bellow, and they are absolutely continuous with respect to each other.

Let $(\mathcal{B}_t)$, $t\geqq 0$, be an increasing family of right continuous sub σ-algebra of $\mathcal{B}$ such that

$$\mathcal{B}_t = \bigcap_{\varepsilon>0} \sigma\{x_s;\, s\leqq t+\varepsilon\} \vee \boldsymbol{Q}\,,$$

where

$$\boldsymbol{Q} = \{A \,|\, \mu^1(A) = 0 \quad \text{or} \quad \mu^1(A) = 1,\ A\in\mathcal{B}\}\,.$$