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SEMIPERFECT MODULES AND QUASI-SEMIPERFECT MODULES

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Recently in his study of QF-2 rings, M. Harada has introduced the extending property of simple modules and the lifting property of simple modules which are mutually dual notions, and he has extensively studied modules with these properties ([10]~[14]). It should be noted that the extending property of simple modules is one of considerable extending properties on modules and it has been somewhat widely studied than the lifting property of simple modules ([11], [13], [14]).

Let M be an R-module and $\mathcal{A}$ a subfamily of the family $\mathcal{L}(M)$ of all submodules of M. M is said to have the extending property of modules for $\mathcal{A}$ provided that every member of $\mathcal{A}$ is embedded to a direct summand of M as an essential submodule. In particular, M is said to have the extending property of simple modules if it has the extending property of modules for $\{A \in \mathcal{L}(M) \mid A \text{ is simple}\}$. Dually M is said to have the lifting property of simple modules if every simple submodule of $M/\mathrm{J}(M)$ is induced from a direct summand of M, where $\mathrm{J}(M)$ is the Jacobson radical of M.

Under this circumstance, the following natural question immediately arises: Can we define the notion dual to 'the extending property of modules for $\mathcal{A}$'? This question seems to be interested in module theory, because this dualization leads us to the dualizations of continuous modules and quasi-continuous modules mentioned in Utumi [28]~[30] and Jeremy [16], [17] (cf. [23]).

Section 1 of this paper is concerned with this problem, and the lifting property of modules for $\mathcal{A}$ is defined as follows: M is said to have the lifting property of modules for $\mathcal{A}$, provided that, for any A in $\mathcal{A}$, there exists a direct summand A^* of M such that $A^* \subseteq A$ and A/A^* is small in M/A^*.

Using this lifting property, in section 2, we introduce $\mathcal{A}$-semiperfect modules and $\mathcal{A}$-quasi-semiperfect modules as duals to $\mathcal{A}$-continuous modules and $\mathcal{A}$-quasi-continuous modules, respectively, which have been studied in [23]. Of course, these names follow from 'semi-perfect module' in the sense of E. Mares [20] defined on projective modules. $\mathcal{L}(M)$-semiperfect modules and $\mathcal{L}(M)$-quasi-semiperfect modules are simply called semiperfect modules