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UNIFORM APPROXIMATION BY ENTIRE FUNCTIONS OF SEVERAL COMPLEX VARIABLES

Dedicated to Professor Yukinari Tôki on his 70th birthday

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Introduction. Let G be a holomorphically convex open subset of $\boldsymbol{C}^n$ and T a closed subset of G. We say that T is *totally real,* if it is the zero set of a non-negative C^2 function ρ which is strictly plurisubharmonic on T. It is known that a real C^1 submanifold M is totally real if and only if it has no complex tangents (cf. [3]). The problem of uniform approximation on totally real submanifols was studied to a great extent by many authors (cf. Wells [9], Hörmander and Wermer [4], Nirenberg and Wells [5], Harvey and Wells [2], [3] and Nunemacher [6]). The result of [6] states that if M is a totally real submanifold then there exists a holomorphically convex open neighborhood B such that every continuous function on M is uniformly approximated on M by functions holomorphic in B. In [8], the author extended this result to the case of totally real sets with C^∞ defining functions. (A totally real set is not necessarily a submanifold. The approximation theorem for totally real sets contains one for totally real analytic subvarieties which was conjectured by Wells [9].)

In this paper, we give a sufficient condition for T and G under which every continuous function on T is uniformly approximated on T by functions holomorphic in G. The theorem we prove contains the following result which is a straight generalization to higher dimensions of Carleman's theorem [1].

Every continuous function on $\boldsymbol{R}^n$, canonically imbedded in $\boldsymbol{C}^n$, is uniformly approximated on $\boldsymbol{R}^n$ by entire functions on n complex variables.

We shall make use of the L^2-method due to Hörmander and Wermer [4] and the swelling method similar to one used in [8].

1. Statements. Let S be a closed subset of an open set U of $\boldsymbol{C}^n$. We denote by $H(S)$ (or $H(S, U)$) the algebra of uniform limits of restrictions of functions holomorphic in a neighborhood of S (or in U, resp.).

We use an abbreviation $L[u; \xi]$ for the Levi form of a C^∞ function u:

$$L[u; \xi] = \sum_{j,k} \frac{\partial^2 u}{\partial z_j \partial \bar{z}_k} \xi_j \bar{\xi}_k, \qquad \xi \in \boldsymbol{C}^n .$$

By an exhaustion function σ of G we mean a positive C^∞ strictly plurisubharmonic