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ON MODULES WITH EXTENDING PROPERTIES

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We have defined the extending property of uniform submodules and of direct sums of independent submodules in [5]. We also have studied modules with lifting property in [4].

In this note, we shall give results dual to those in [4] for the extending properties. Finally, we shall give the completely forms of modules with extending property of uniform submodules over a Dededind domain.

1 Definitions

Throughout this paper we assume that a ring R has the identity element and every module M is a unitary right R-module. We recall here defintions in [5].

If $\mathrm{End}_R(M)$ is a local ring, we call M a *completely indecomposable.* We denote the *socle* and an *injective envelope* of M by $S(M)$ and $E(M)$, respectively. Let $T=\sum_K \oplus T_\alpha$. If a submodule L of T is contained in $\sum_J \oplus T_\alpha$ for some finite subset J of K, we say L is *finitely contained* (briefly f.c.) (with respect to $\sum_K \oplus T_\alpha$). It is clear that this defintion depends on the direct decomposition of T. We have studied a cyclic hollow module in [3]. We note that the concept dual to a cyclic hollow module is a uniform module with non-zero socle.

If a submodule N of M is essential in M, we indicate it by $M_e\supseteq N$. Let $\{C_\gamma\}_I$ be set of independent submodules with certain property (*). If there exists a set of independent submodules $\{N_\gamma\}_I$ such that $N_{\gamma e}\supseteq C_\gamma$ for all $\gamma\in I$ and $\sum_I \oplus N_\gamma$ is a direct summand of M, we say *the direct sum of* $\{C_\gamma\}_I$ *with* (*) *is essentially extended to a direct summand of* M. If every direct sum of independent submodules with (*) is essentially extended to a direct summand of M, then we say M has *the extending property of direct sums of independent submodules with* (*). Especially, if $S(M)=\sum_I \oplus C_\gamma$ and $M=\sum_I \oplus N_\gamma$ in the above, we say M has *the extending property of direct decompositions of* $S(M)$. Next, we consider a case of $|I|$ (=the cardinal of I)=1. In this case we say M has *the extending property of submodules with* (*).

In order to get good results, we always assume T_1 *is completely indecomposable* in the above when $|I|=1$ and C_1 is uniform.