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FINITE GROUPS ADMITTING AN AUTOMORPHISM OF PRIME ORDER FIXING A CYCLIC 2-GROUP

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1. Introduction

In this paper, we shall give a proof of the following Theorem, which is a conjecture of B. Rickman [9]; in special case, $C_G(\phi)$ has order 2, M.J. Collins and B. Rickman proved in [2].

Theorem. *Let G be a finite group which admits an automorphism ϕ of odd prime order r whose fixed-point-subgroup $C_G(\phi)$ is a cyclic 2-group. Then G is solvable.*

All groups considered in this paper are assumed finite. Our notation corresponds to that of Gorenstein [7].

An important tool that is brought to attack the problem is B. Baumann's classification of finite simple groups whose Sylow 2-subgroups are maximal [1], and in analogy with Matsuyama [8] that used the results of [1], we shall prove that $\mathcal{N}_G(S;2)\neq 1$, where S is a ϕ-invariant Sylow 3-subgroup of G.

C.A. Rowley has obtained a proof of the theorem under the additional hypothesis that G does not involve S_4, the symmetric group on 4 letters.

The Theorem is a contribution to the continuing problem of showing that finite groups which admit an automorphism ϕ of odd prime order such that $C_G(\phi)$ is a 2-group are solvable.

2. Preliminaries

We first quote some frequently used results.

2.1. (Thompson [12])

Let G be a group which admits a fixed-point-free automorphism of prime order. Then G is nilpotent.

2.2. (Rowley [10])

Let G be a solvable group admitting an automorphism of odd prime order p such that $C_G(\phi)$, the fixed-point-subgroup of ϕ in G, is a cyclic q-group, $q\neq p$. Then, for any prime r, G is either r-nilpotent or r-closed.