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GENERALIZATION OF A THEOREM OF PETER J. CAMERON

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Peter J. Cameron [3] has shown that a primitive permutation group G has rank at most 4 if the stabilizer G_α of a point α is doubly transitive on all its nontrivial suborbits except one.

The purpose of this paper is to prove the following two theorems, one of which extends the Cameron's result.

Theorem 1. *Let G be a primitive permutation group on a finite set* Ω, *and all nontrivial G-obrits in Cartesian product* $\Omega\times\Omega$ *be* $\Gamma_1, \cdots, \Gamma_s, \Delta_1, \cdots, \Delta_t$, *where* G_α *is doubly transitive on* $\Gamma_i(\alpha)=\{\beta\,|\,(\alpha, \beta)\ni\Gamma_i\}$, $1\leqq i\leqq s$ *and not doubly transitive on* $\Delta_i(\alpha)$, $1\leqq i\leqq t$. *Suppose that G has no subdegree smaller than* 4 *and that* $t>1$. *Then, we have*

$$s\leqq 2t-r\,,$$

where $r=\#\{\Delta_i\,|\,\Delta_i=\Gamma_j^*\circ\Gamma_j\;1\leqq j\leqq s\}$. *Moreover if* $r=1$, *then we have*

$$s\leqq 2t-2\,.$$

(For the notation $\Gamma_j^*\circ\Gamma_j$, see the section 1)

Theorem 2. *Under the hypothesis of Theorem* 1, *if* $r=t$, *then* $s=t=2$, *and G is isomorphic to the small Janko simple group and* G_α *is isomorphic to* $PSL(2, 11)$.

For the case of $t\geqq 3$, I don't know the example satisfying the equality $s=2t-r$, and when $r=1$, the exampel satisfying the equality $s=2t-2$. I know only three examples with $t=2$ and $s=2$.

The small Janko simple group J_1 of order 175560 has a primitive rank 5 represestation of degree 266 in which the stabilizer of a point is isomorphic to PSL (2, 11) and acts doubly transitively on suborbits of lengths 11 and 12; the other suborbit lengths are 110 and 132 (See Livingstone [7]). The Mathieu group M_{12} has a primitive rank 5 representation of degree 144 in which the stabilizer of a point is isomorphic to PSL (2, 11) and acts doubly transitively on two suborbits of length 11; the other suborbit lengths are 55 and 66 (See Cameron [4]).