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SYMMETRIC GROUPOIDS

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Introduction

Loos has shown in [3] that a symmetric space can be defined as a manifold carrying a diffeomorphic binary operation that satisfies three algebraic and one topological condition. This algebraic approach to symmetric spaces has been explored by Loos in [4], and by various other workers, for example Kikkawa in the series of papers [2]. Abstracting the algebraic properties of a symmetric space, Nobusawa introduced in [6] the concept of symmetric structure on a set. In that paper, and a sequel to it [1], the structure of finite symmetric sets satisfying a certain transitivity condition has been invesitgated. In particular, it was shown in [1] that there is a close relationship between symmetric sets and groups that are generated by involutions.

The purpose of this paper is to lay the foundations of a general theory of symmetric sets. The principal emphasis of this program is the connection between symmetric sets and groups that are generated by involutions. For the most part, we use the resources of group theory to gain insight into the structure of symmetric sets. It is to be hoped that in the future the flow of ideas will move the other way.

Our viewpoint in this paper is influenced by the ideas of universal algebra and category theory. Symmetric sets are looked upon as members of a particular variety of groupoids. For this reason, it seems appropriate to break a tradition by using the term "symmetric groupoid" rather than "symmetric set." Henceforth, this convention will be followed. Also, we will use the abbreviation "*GI* Group" for a group that is generated by the set of its involutions. Other than these idiosyncrasies our terminology in the paper is generally standard.

A brief outline of this work follows. The first section introduces the principal concepts that form the subject of the paper. Standard notation is established, and a few elementary facts are noted. Section two is devoted to categorical matters. Special kinds of morphisms of symmetric groupoids and *GI* groups are introduced in such a way that the natural correspondence between

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