Iwasaki, C.
Osaka J. Math.
14 (1977), 569–592

THE FUNDAMENTAL SOLUTION FOR PSEUDO-DIFFERENTIAL OPERATORS OF PARABOLIC TYPE

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(Received October 20, 1976)

Introduction

In this paper we shall construct the fundamental solution $E(t, s)$ for a degenerate pseudo-differential operator L of parabolic type only by symbol calculus and, as an application, we shall solve the Cauchy problem for L:

$$\left\{\begin{array}{ll} Lu(t)=f(t) & \text{in } t>s\,, \\ u(s)=u_0\,. & \end{array}\right. \tag{0.1}$$

Another application of the present fundamental solution will be done in [12] in order to construct left parametrices for degenerate operators studied by Grushin in [2].

Now let us consider the operator L of the form

$$L=\frac{\partial}{\partial t}+p(t;x,D_x)\,,$$

where $p(t; x, D_x)$ is a pseudo-differential operator of class $S^m_{\lambda,\rho,\delta}$ with a parameter t $(\rho>\delta)$ (See §1). For the operator $p(t; x, D_x)$ we set the following conditions:

$$\operatorname{Re} p(t;x,\xi)+c_0\geqslant c_1\lambda(x,\xi)^{m'} \tag{0.2}$$

$$|p^{(\alpha)}_{(\beta)}(t;x,\xi)/(\operatorname{Re} p(t;x,\xi)+c_0)|\leqslant C_{\alpha,\beta}\lambda(x,\xi)^{-(\rho,\alpha)+(\delta,\beta)}\,, \tag{0.3}$$

where $m\geqslant m'\geqslant 0$ and $\lambda=\lambda(x,\xi)$ is a basic weight function defined in §1. We note that $\lambda(x, \xi)$ in general varies even in x and increases in polynomial order.

We call $E(t, s)$ a fundamental solution for L when $E(t, s)$ satisfies

$$\left\{\begin{array}{ll} LE(t,s)=0 & \text{in } t>s\,, \\ E(s,s)=I\,. & \end{array}\right.$$

The main theorem of this paper is stated as follows.

Main theorem. *Under the conditions* (0, 2) *and* (0, 3) *we can construct the unique fundamental solution* $E(t, s)$ *for* L *as a pseudo-differential operator of*