

QUANTITIES AND REAL NUMBERS

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Introduction

The theory of real numbers, as a basis of mathematical analysis, had been already completed in the nineteenth century in several ways (cf. [1], [2], [3]), and now we seem to have nothing to do newly with it. These mathematical theories have been established as the completion of the system of rational numbers, while the intimate relation between the quantity and the number has been rather neglected.

Here we shall start from the characterization of the system of positive quantities and derive the system of positive real numbers as the set of automorphisms of the system of positive quantities. Then, the extension of the system of positive real numbers to the whole system of real numbers can be easily carried out.

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1. System of positive quantities

A. Axioms of the system of positive quantities. Let Q be a system of quantities of the same kind with the following properties with respect to the addition:

- (I₁) If a and $b \in Q$, then $a+b \in Q$ ($a+b$ is uniquely determined).
- (I₂) $a+b = b+a$ if a and $b \in Q$.
- (I₃) $(a+b)+c = a+(b+c)$ ($a, b, c \in Q$).
- (I₄) If $a+c = b+c$ ($a, b, c \in Q$), then $a = b$.

The system of quantities Q is said to be *positive*, if the following conditions are fulfilled:

- (II₁) If a and $b \in Q$, then $a+b \neq a$.

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