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SUPPLEMENTARY REMARKS ON CATEGORIES OF INDECOMPOSABLE MODULES

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In the previous papers [3], [4], we have defined a full sub-category $\mathfrak{A}$ in the category $\mathfrak{M}_R$ of modules over a ring R, whose objects consist of injective modules or directsums of completely indecomposable modules.

Making use of those ideas, in this short note, we shall give a proof of Z. Papp's theorem in [9] as an application of [3], Theorem 1 and generalize Theorems 4 and 7 in [4] to cases of semi-T-nilpotent system and quasi-projective module, respectively. Especially, we shall show that if R is a right perfect ring, then every quasi-projective module is a directsum of completely indecomposable modules and the Krull-Remak-Schmidt's theorem is valid for those direct decompositions.

In this note, we always assume that the ring R has the identity and every module is an unitary R-module. We shall use the same notations and definitions in [3], [4] and [5] for categories, those in [1] and [8] for semi-perfect modules and those in [5] for quasi-projective modules.

1. Papp's theorem

We shall give an application of [3], Theorem 1.

Theorem 1 ([9], Z. Papp). *Let R be a ring. If every* (*right*) *R-injective module is a directsum of indecomposable modules, then R is* (*right*) *noetherian.*

Proof. It is known by [2], Proposition 4.1 that R is noetherian if and only if any directsum of injective modules is also injective. Let $\mathfrak{A}$ be the full sub-category of all injective R-modules in the category of right R-modules and $\mathfrak{J}$ the Jacobson radical of $\mathfrak{A}$. Then $\mathfrak{A}/\mathfrak{J}$ is a completely reducible C_3-abelian category by the assumption and [3], Theorem 1. Let $\{Q_i\}_1^\infty$ be a family of injective modules, and E an injective hull of $\sum\oplus Q_i$. From the assumption $E=\sum\oplus E_i$, where E_i's are (completely) indecomposable. Hence, $\sum\oplus E_j=\sum\oplus Q_i$ in $\mathfrak{A}/\mathfrak{J}$ by [3], Theorem 1. Therefore, $E\approx\sum\oplus Q_i$, which means that $\sum\oplus Q_i$ is injective. Hence, R is noetherian.