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AN INTEGRO-DIFFERENTIAL EQUATION FOR A COMPOUND POISSON PROCESS WITH DRIFT AND THE INTEGRAL EQUATION OF H. CRAMER

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1. Introduction

Let $(Y(t))_{t\geq 0}$ be a compound Poisson process on $\mathbf{R}=(-\infty, \infty)$ with the characteristic function

$$\mathbf{E}(e^{iyY(t)}) = \exp\left\{t\int_{-\infty}^{\infty}(e^{iyu}-1)\,\nu(du)\right\}, \tag{1.1}$$

where ν is a finite measure. For short we assume that

$$\int_{-\infty}^{\infty}\nu(du) = 1\,. \tag{1.2}$$

Let $(X(t))_{t\geq 0}$ be the compound Poisson process with a drift term at $(a\in\mathbf{R})$:

$$X(t) = at+Y(t)\,. \tag{1.3}$$

It is known that, if f is a bounded function with a second continuous derivative, $t^{-1}[\mathbf{E}\{f(x+X(t))\}-f(x)]$ converges uniformly on every bounded intervals to

$$Af(x):= af'(x)+\int_{-\infty}^{\infty}[f(x+y)-f(x)]\,\nu(dy)\,. \tag{1.4}$$

One now enlarges the domain of A as follows. Let G be an open subset of $\mathbf{R}$. Denote by $\mathcal{B}$ the class of all bounded, measurable and real-valued functions on $\mathbf{R}$. Define

$$\begin{aligned}\mathcal{D}(A;G):&=\{f\in\mathcal{B};\ f \text{ is absolutely continuous in } G\} \qquad \text{if } a\neq 0,\\ &=\mathcal{B} \qquad \text{if } a=0.\end{aligned} \tag{1.5}$$

For $f\in\mathcal{D}(A;G)$, Af is defined almost everywhere in G by (1.4).

The main result of this note is Theorem 2 in section 2 which describes, for each $\lambda\geq 0$, a natural class of functions in $\mathcal{D}(A;G)$ satisfying the equation

$$(\lambda-A)f = 0 \qquad \text{almost everywhere in } G. \tag{1.6}$$