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CONTRIBUTIONS TO THE THEORY OF INTERPOLATION OF OPERATIONS

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1. Introduction. Let (R, μ) and (S, ν) be two measure spaces of totally σ-finite in the sense of P. Halmos [7]. Let us consider operation T which transforms measurable functions on R to those on S. The operation T is called quasi-linear if:

(i) $T(f_1+f_2)$ is uniquely defined whenever Tf_1 and Tf_2 are defined and

$$|T(f_1+f_2)| \leqq \kappa(|Tf_1|+|Tf_2|)$$

where κ is a constant independent of f_1 and f_2;

(ii) $T(cf)$ is uniquely defined whenever Tf is defined
and

$$|T(cf)| = |c| \ |Tf|$$

for all scalars c.

We say that

$$\tilde{f} = Tf$$

is an operation of type (a, b), $1 \leqq a \leqq b \leqq \infty$, if :

(i) Tf is defined for each $f \in L^a_\mu(R)$, that is for each f measurable with respect to μ such that

$$||f||_{a,\mu} = \left(\int_R |f|^a d\mu\right)^{1/a}$$

is finite, the right side being interpreted as the essential upper bound (with respect to μ) of $|f|$ if $a = \infty$;

(ii) for every $f \in L^a_\mu(R)$, $\tilde{f} = Tf$ is in $L^b_\nu(S)$ and

$$||\tilde{f}||_{b,\nu} \leqq M \, ||f||_{a,\mu}, \tag{1.1}$$

where M is a constant independent of f.
The least admissible value of M in (1.1) is called the (a, b)–norm of operation T.