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NOTE ON A THEOREM DUE TO MILNOR

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1. Introduction

J. Milnor [1] has proved the following theorem: Let N be a closed topological manifold which is a mod 2 homology n-sphere, and T be a fixed point free involution on N. Then, for every continuous map $f:N\to N$ such that f_*: $H_n(N; \boldsymbol{Z}_2)\to H_n(N; \boldsymbol{Z}_2)$ is not trivial, there exists a point $y\in N$ such that $fT(y)=Tf(y)$.

In the present paper, we shall show that this result can be generalized as follows:

Theorem 1. *Let N and M be topological n-manifolds on each of which there is given a fixed point free involution T ($n\geqq 1$). Assume that N has the mod 2 homology of the n-sphere. Then, for every continuous map $f:N\to M$ such that f_*: $H_n(N; \boldsymbol{Z}_2)\to H_n(M; \boldsymbol{Z}_2)$ is not trivial, there exists a point $y\in N$ such that $fT(y)=Tf(y)$.*

Our method is different from Milnor [1], and we shall apply the method we used in [2] to prove a generalization of Borsuk-Ulam theorem.

REMARK. The theorem is regarded in some sense as a converse of Corollary 1 of the main theorem in [2].

Throughout this paper, all chain complexes and hence all homology and cohomology groups will be considered on $\boldsymbol{Z}_2$.

2. The chain map

Let Y be a Hausdorff space on which there is given a fixed point free involution T. Denote by π the cyclic group of order 2 generated by T. We shall denote by Y_π the orbit space of Y, and by p: $Y\to Y_\pi$ the projection. Consider the induced homomorphisms $T_\sharp:S(Y)\to S(Y)$ and $p_\sharp:S(Y)\to S(Y_\pi)$ of singular complexes. Then a chain map

$$\phi: S(Y_\pi)\to S(Y)$$

can be defined by

$$\phi(c)=\tilde{c}+T_\sharp(\tilde{c}),\quad p_\sharp(\tilde{c})=c,$$