Adachi, M. Osaka J. Math. 3 (1966), 121-137

# NOTE ON MICROBUNDLES\*

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#### (Received February 23, 1966)

In [2] Lashof and Rothenberg have defined the css-group 0 and the Kan complex PD, and shown a certain exact sequence of abelian groups (Theorem (4.2)) which is fundamental to the studies of the PL-microbundles and smoothing.

In the present note we shall define a css-group H for the topological microbundles parallel to the css-group PL for the PL-microbundles (§ 1), and show an analogous exact sequence of abelian groups (§ 4) which seems to have some meaning to the study of the topological microbundles (§ 2, § 3).

Our method is quite analogous to that of Lashof and Rothenberg [2], and Milnor [3], and is based on Heller's theory [1].

The author is grateful to Professors R. Shizuma, K. Shiraiwa and T. Nakamura for their kind criticisms.

### 0. Preliminaries

#### a) Directed systems of css-complexes.

Let  $\Sigma$  be a partially ordered set, i.e. a set in which we have a transitive relation < defined for some (but not necessary all) pairs of elements.  $\Sigma$  is called a directed set if every pair of elements has a common successor: given  $\sigma$  and  $\tau$  in  $\Sigma$  there is an element  $\rho$  in  $\Sigma$  satisfying  $\sigma < \rho$  and  $\tau < \rho$ .

In the present note all css-complexes are supposed to satisfy Kan's extension condition unless otherwise stated.

Suppose to each element  $\sigma$  of  $\Sigma$  is assigned a css-complex<sup>1</sup>)  $K_{\sigma}$  (cssgroup  $G_{\sigma}$ ) and to each pair of elements  $\sigma < \tau$  of  $\Sigma$  there corresponds a css-map  $h_{\sigma\tau}$  of  $K_{\sigma}$  into  $K_{\tau}$  (css-homomorphism  $h_{\sigma\tau}$  of  $G_{\sigma}$  into  $G_{\tau}$ ) such that if  $\rho < \sigma < \tau$  then

<sup>\*)</sup> This work is partially supported by Yukawa Fellowship.

<sup>1)</sup> For the theory of css-complexes, see for example Heller [1], Moore [5], Puppe [6].