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AN APPLICATION OF FUNCTIONAL HIGHER OPERATION

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Introduction

Let ${}^1M^n$ be the Moore space $M(n, Z_p)$ (i.e., a simply connected space with two non-vanishing homology groups $H_0({}^1M^n; Z)=Z$ and $H_n({}^1M^n; Z)=Z_p$), where p is an *odd* prime. Let ${}^1\pi_i$ be the stable homotopy group $\lim\ [{}^1M^{n+i}; {}^1M^n]$, and ${}^1\pi_*=\sum_i {}^1\pi_i$. Then, there are non-trivial elements $\alpha\in{}^1\pi_{2p-2}$ and $\beta_1\in{}^1\pi_{2p(p-1)-1}$[9].

Let ${}^2M^n$ be the mapping cone of α (i.e., ${}^2M^n={}^1M^n\cup_\alpha T{}^1M^{n+2p-2}$ for sufficiently large n), and ${}^2\pi_i$ be the stable homotopy group $\lim\ [{}^2M^{n+i}; {}^2M^n]$, ${}^2\pi_*=\sum_i 2\pi_i$. Corresponding to $\beta_1\in{}^1\pi_{2p(p-1)-1}$, we can define a non-trivial element $\beta\in{}^2\pi_{2p^2-2}$.

Then, our main theorem is

Theorem. *$\alpha^t\neq 0$ in ${}^1\pi_*$ and $\beta^t\neq 0$ in ${}^2\pi_*$ for all $t\geqq 1$.*

This paper is divided into three chapters. In the first chapter, we deal with the functionalization of Adams-Maunder higher cohomology operations [1], [3], and study some relations among them; in chapter 2, suitable chain complexes are constructed by means of the Milnor basis of the mod p Steenrod algebra [4]. In the last chapter, the main theorem is proved in a slightly general form using the results in preceding chapters, especially Proposition 4. 3.

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CHAPTER 1. FUNCTIONAL OPERATIONS

1. Preliminaries

In this paper, spaces are arcwise connected, based and having the homotopy type of a CW-complex. Maps take base point to base point