Kuroda, S.
Osaka J. Math.
41 (2004), 759–792

A FINITE UNIVERSAL SAGBI BASIS FOR THE KERNEL OF A DERIVATION

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(Received April 11, 2003)

1. Introduction

To find and to calculate generating sets for invariant rings is a fundamental problem in invariant theory with a long tradition. With the progress of computers, the significance of computational methods in this field has increased. The SAGBI bases are the sets of generators of a subalgebra of a polynomial ring which have certain computational property. These are the natural "Subalgebra Analogue to Gröbner Bases for Ideals" introduced at the end of 1980's by Robbiano and Sweedler [20] and Kapur and Madlener [8], independently. There are indeed some applications of the SAGBI bases to invariant theory. The algorithm of Stillman and Tsai [23] gives a method for computing generating sets for certain invariant rings by using this notion. However, compared with the theory of Gröbner bases, that of SAGBI bases has made a slow progress, and many basic problems remaining unsolved. The purpose of this paper is to investigate the properties of a SAGBI basis for the kernel of a derivation on a polynomial ring.

The kernel of a derivation on a polynomial ring is closely related to an invariant ring. It is an important object in the study of invariant theory and the fourteenth problem of Hilbert. It is well-known that some kind of derivation corresponds to an action of one-dimensional additive group, and the kernel and the invariant subring are the same. Moreover, various counterexamples to the fourteenth problem of Hilbert can be described as the kernel of a derivation. Nagata's counterexample [17] and Roberts' counterexample [22] were described as this by Derksen [2] and by Deveney and Finston [4], respectively. Nowicki showed that the invariant subring for a linear action of a connected linear algebraic group on a polynomial ring is obtained as the kernel of a derivation [18]. Recently, new counterexamples to the fourteenth problem of Hilbert were constructed by using the kernel of a derivation by several people (cf. [1], [6], [10], [13]). We believe that a computational methods will give us further progress in this field.

In this paper, k is always a field of characteristic zero except Section 6. Let

Partly supported by the Grant-in-Aid for JSPS Fellows, The Ministry of Education, Science, Sports and Culture, Japan.